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Robustness Analysis of Time Petri Nets

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Context: Verifying Complex Timed Systems (1/2)

- Need for early bug detection
  - Bugs discovered when final testing: expensive
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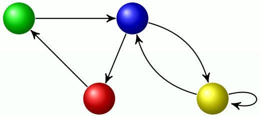
- Question: does the system behave well?

Context: Verifying Complex Timed Systems (2/2)

- Use formal methods



A finite model of the system

$AG\neg$ 

A formula to be satisfied

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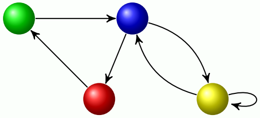

$\overset{?}{\models}$

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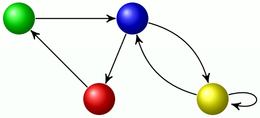

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Yes

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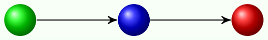

Counterexample

Motivation: Robustness Analysis

- Timed systems are characterized by a set of timing constants
  - "The packet transmission lasts for $50\,ms$"
  - "The sensor reads the value every $10\,s$"
- Verification for one set of constants does not guarantee the correctness for other values
- Challenge: Robustness [Markey, 2011]
  - What happens if 50 is implemented with 49.99?
  - Until which value can we increase or decrease 50 such that the system still behaves well?

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- Challenge: Robustness [Markey, 2011]
  - What happens if 50 is implemented with 49.99?
  - Until which value can we increase or decrease 50 such that the system still behaves well?
- Parametric analysis
  - Consider that timing constants are parameters
  - Find good values for the parameters, such that the system still behaves well

Outline

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1. Parametric Inhibitor Time Petri Nets
2. Robustness Analysis Using the Inverse Method
3. Perspectives

Petri Nets [Petri, 1962]

- Advantages of Petri nets
  - Detailed view of the process with an expressive graphical representation based on places and transitions
  - A formal semantics
  - Powerful model checking tools

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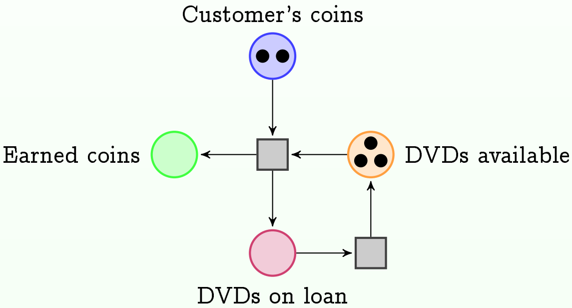

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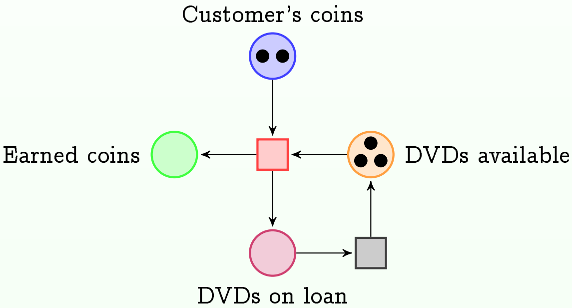

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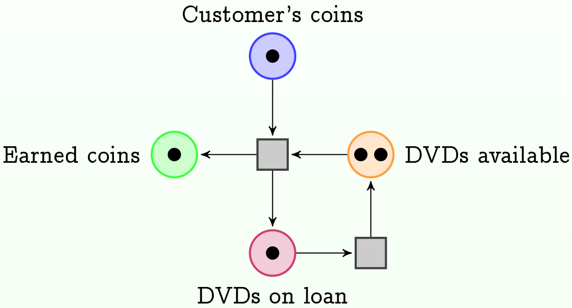

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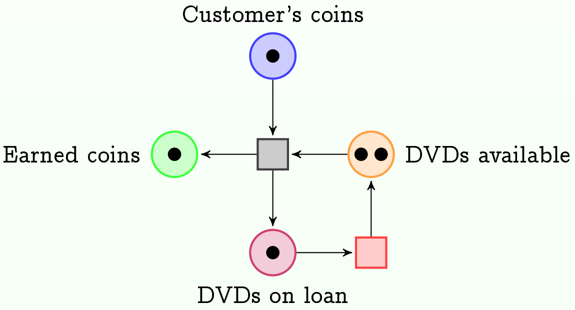

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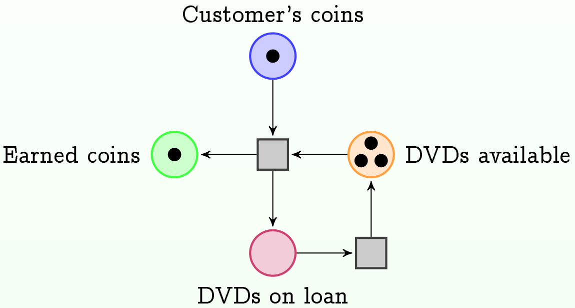

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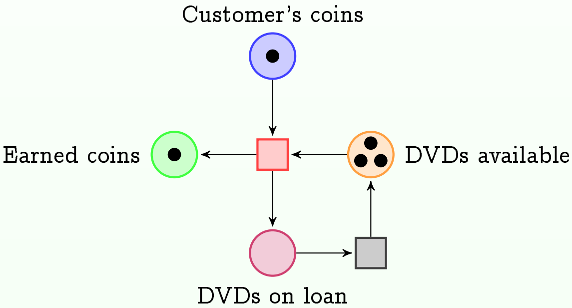

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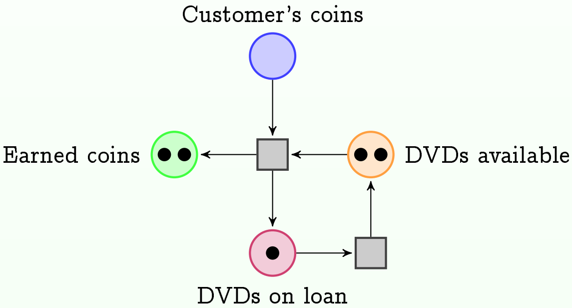

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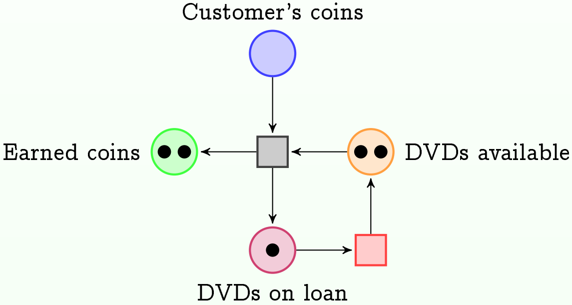

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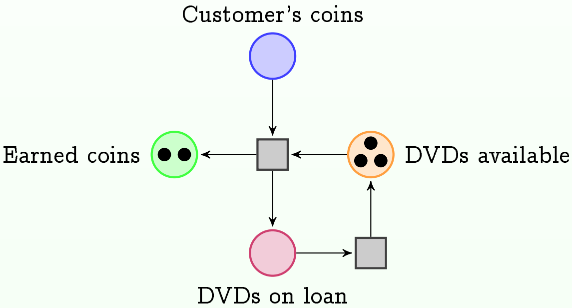
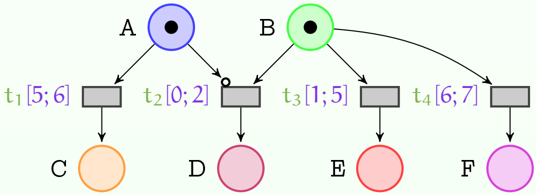


Time Petri Nets With Inhibitor Arcs

- Powerful formalism for verifying real-time systems [Merlin, 1974]
- Transition t_1 can be fired from 5 to 6 units of time after it is enabled
- An enabled transition must fire before (or at) its upper bound
  - Except if another transition fires before
- An inhibitor arc (t_2) enables its transition once its predecessor place (A) is empty



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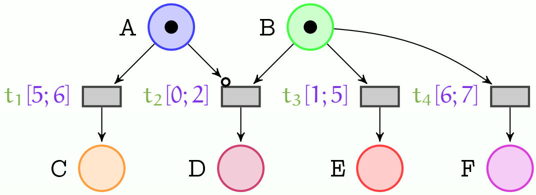

Some possible runs

AB

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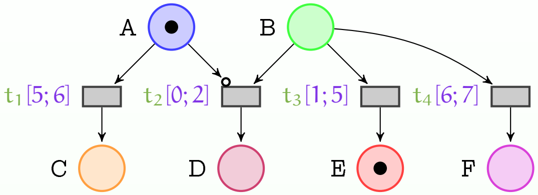

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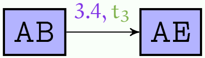

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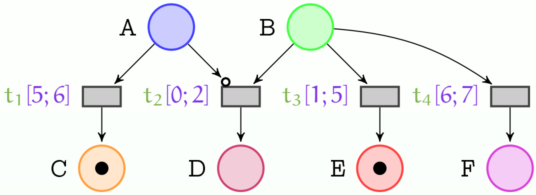

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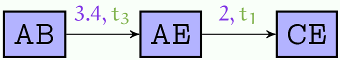

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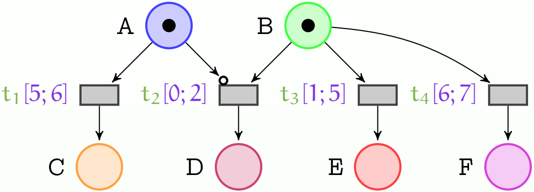

Some possible runs

$$\boxed{AB} \xrightarrow{3.4,\ t_3} \boxed{AE} \xrightarrow{2,\ t_1} \boxed{CE}$$

$$\boxed{AB}$$

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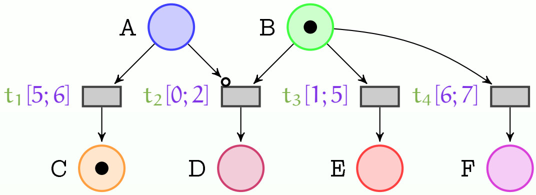

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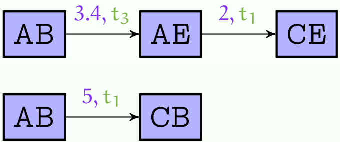

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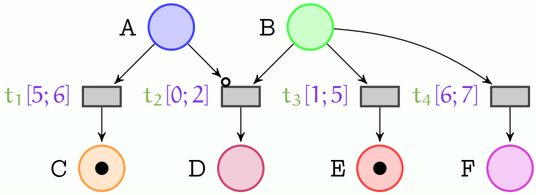

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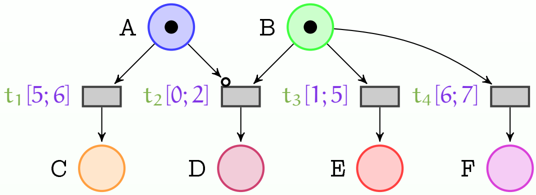

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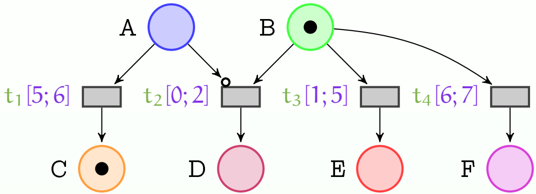

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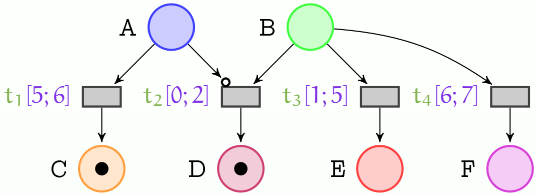

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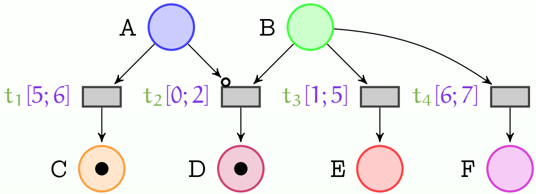

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Set of traces

$AB \xrightarrow{t_3} AE \xrightarrow{t_1} CE$

$AB \xrightarrow{t_1} CB \xrightarrow{t_3} CE$

$AB \xrightarrow{t_1} CB \xrightarrow{t_2} CD$

Trace: time-abstract behavior

Objectives

- We consider that the system behavior (good or bad) depends on the traces
- Questions
  - Until which value can we minimize the upper bound of t_3 (5) so that the system behavior remains the same?
  - Can we quantify the system robustness?

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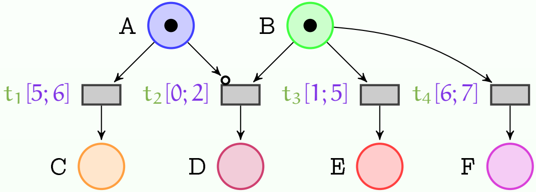
- Questions
  - Until which value can we minimize the upper bound of t_3 (5) so that the system behavior remains the same?
  - Can we quantify the system robustness?
- Idea
  - Reason with parametric time Petri nets
  - Synthesize a constraint on the parameters that guarantees the same behavior

Parametric Time Petri Nets

- Constants in firing intervals replaced with parameters [Traonouez et al., 2009]



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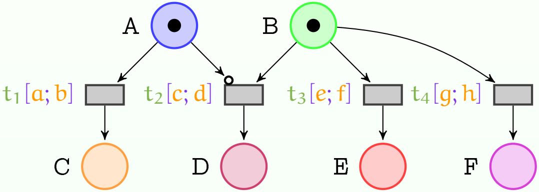

Outline

The Inverse Method

- Input
  - A PITPN $\mathcal{P}$
  - A reference valuation π_0 of all the parameters of $\mathcal{P}$

π_0

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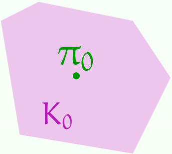
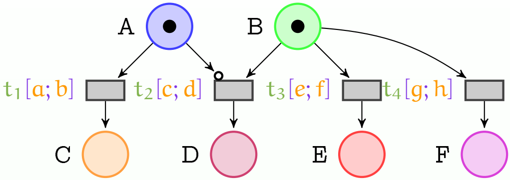
- Input
  - A PITPN $\mathcal{P}$
  - A reference valuation π_0 of all the parameters of $\mathcal{P}$
- Output: K_0
  - Convex constraint on the parameters such that
    - $\pi_0 \models K_0$
    - For all points $\pi \models K_0$, $\mathcal{P}[\pi]$ and $\mathcal{P}[\pi_0]$ have the same trace sets



The Inverse Method: General Idea

- Initially defined for timed automata [André et al., 2009]
- Extended to PITPNs [André and Garg, 2012]
- The idea
  - Exploration of the parametric state space
  - Instead of negating bad states (as in "CEGAR" approaches), remove π_0-incompatible states
  - Return the intersection of all constraints on the parameters

Application to an Example



π_0

$a = 5 \quad b = 6$

$c = 0 \quad d = 2$

$e = 1 \quad f = 5$

$g = 6 \quad h = 7$

- Forward analysis

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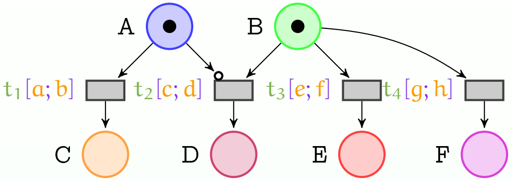

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AB	
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$e \leq f$	$g \leq h$

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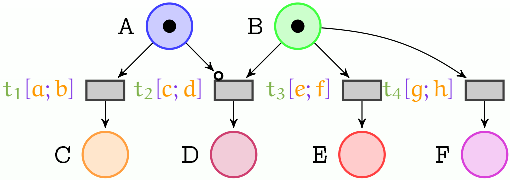

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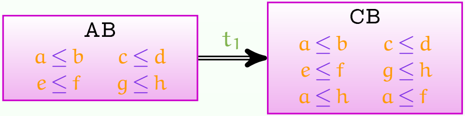

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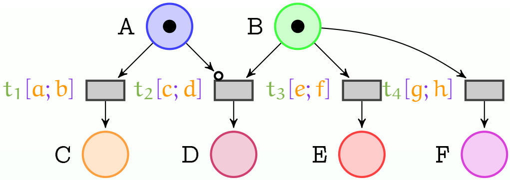

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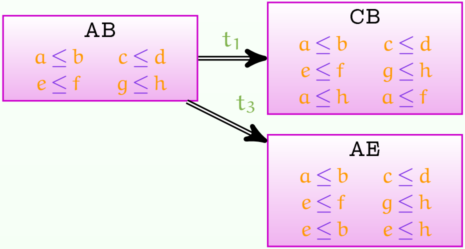

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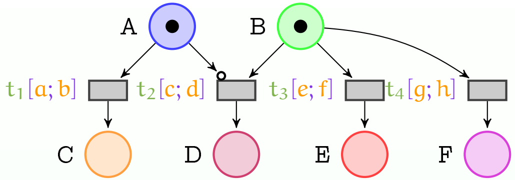

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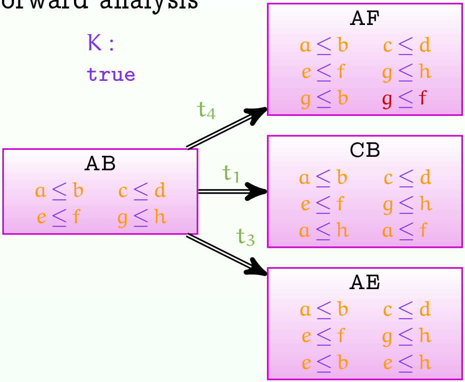

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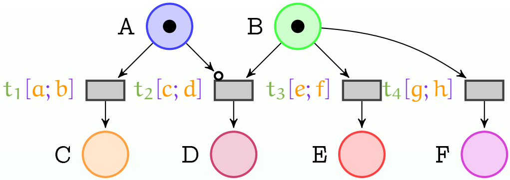

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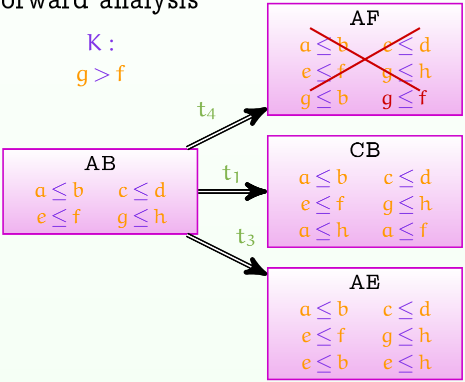

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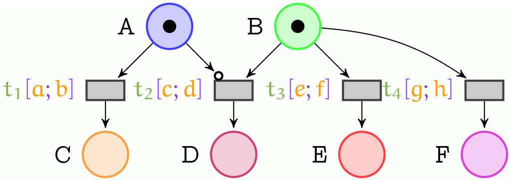

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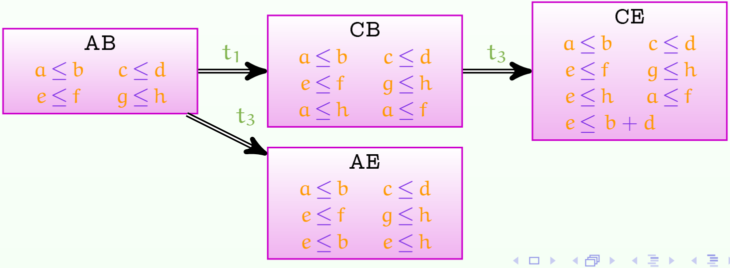

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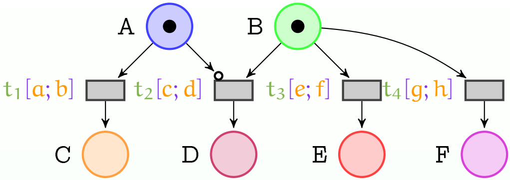

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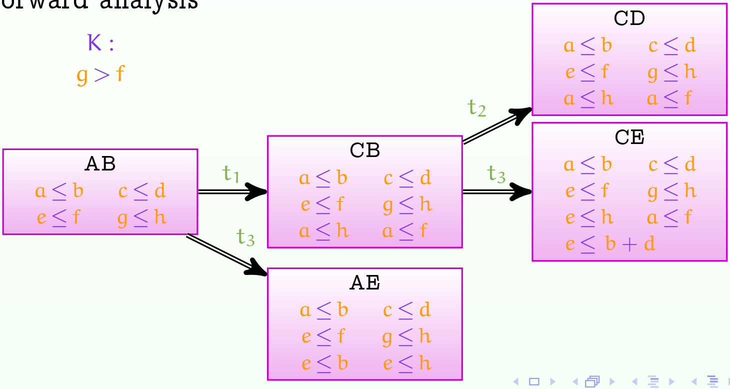

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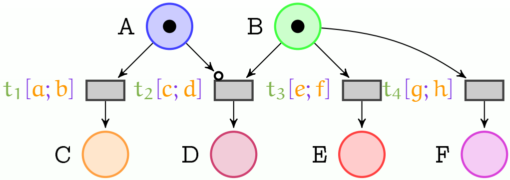

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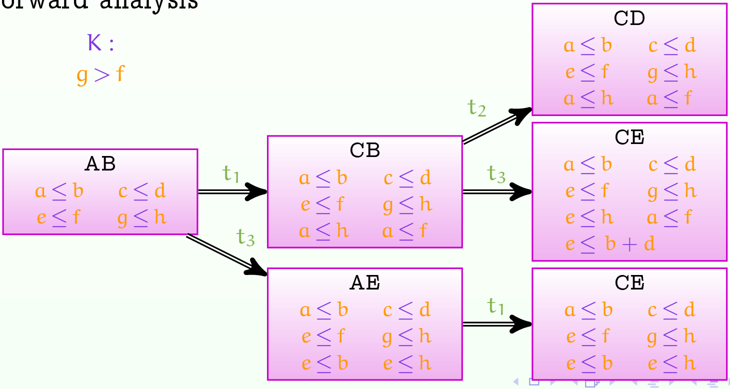
$g = 6 \quad h = 7$

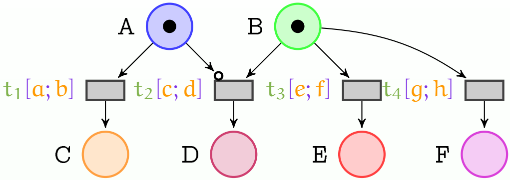
- Forward analysis

$K:$

$g > f$



Application to an Example



π_0

$a = 5 \quad b = 6$

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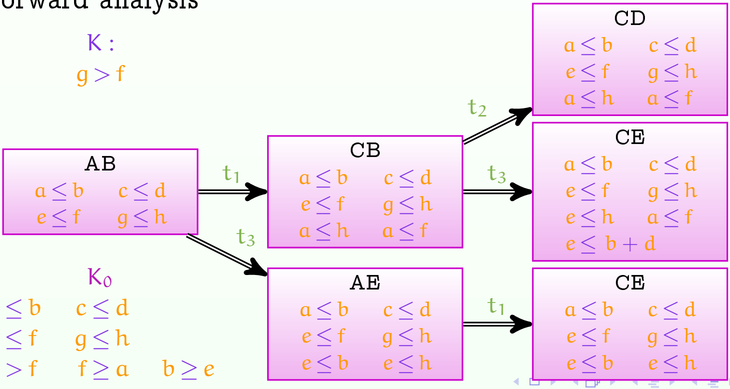

K_0

$a \leq b \quad c \leq d$

$e \leq f \quad g \leq h$

$g > f \quad f \geq a \quad b \geq e$

Application to an Example: Interpretation

- Resulting constraint K_0

$$a \leq b \quad c \leq d \quad e \leq f \quad g \leq h$$
$$g > f \quad f \geq a \quad b \geq e$$

- Interpretation
  - For any $\pi \models K_0$, the trace set is the same as for π_0

- Remark
  - c and d do not appear within K_0 (except $c \leq d$): for any $\pi \models K_0$, the values of c and d do not influence the trace set

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- Application
  - Until which value can we minimize the upper bound f of t_3 (5) so that the system behavior remains the same?
  - Due to $f \geq a$ with $a = f = 5$, one cannot decrease f
    $\rightsquigarrow$ The system is not robust w.r.t. small variations of f or a

Correctness

Theorem (Correctness)

Let $\mathcal{P}$ *be a PITPN, and* π_0 *be a reference valuation.*
Let $K_0 = IM(\mathcal{P}, \pi_0)$. *Then:*

1. $\pi_0 \models K_0$ *and*
2. $\forall \pi \models K_0$, $\mathcal{P}[\pi]$ *and* $\mathcal{P}[\pi_0]$ *have the same trace set.*

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Proof.

By induction on the length of the runs. □

Advantages

- Quantification of the system robustness
- Allows timing optimizations
- Allows the replacement of a component with another one
  - As long as the new timings satisfy K_0

Outline

Perspectives (1/2)

- Extension to colored Petri nets [Jensen and Kristensen, 2009]
  - Tokens and places have a type (“color set”)
  - Arcs are labeled with expressions
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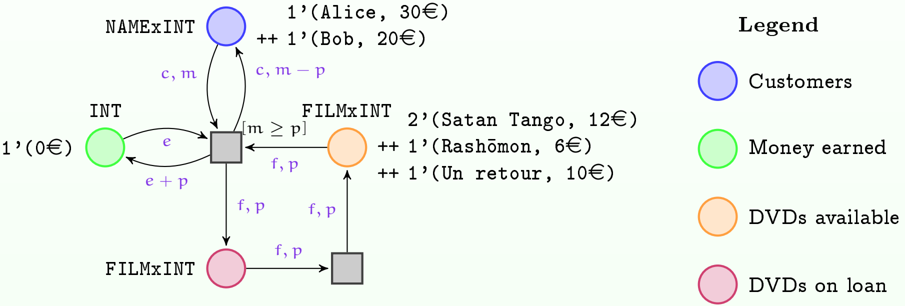

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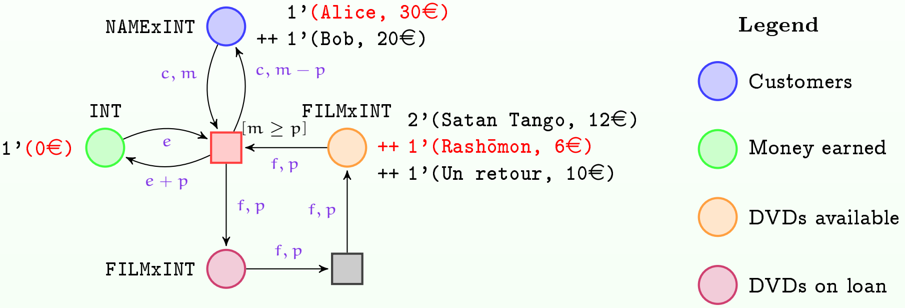

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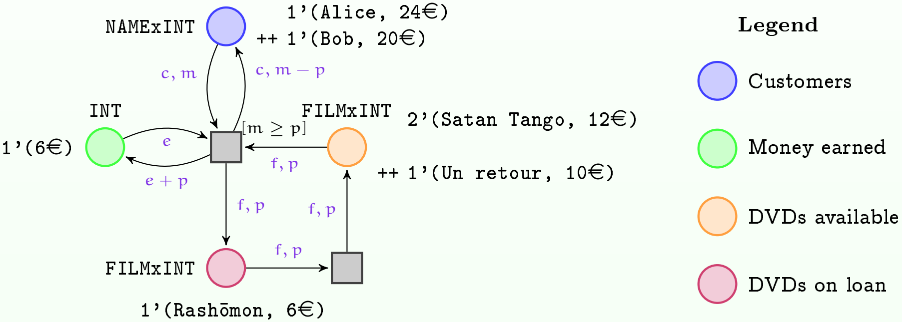

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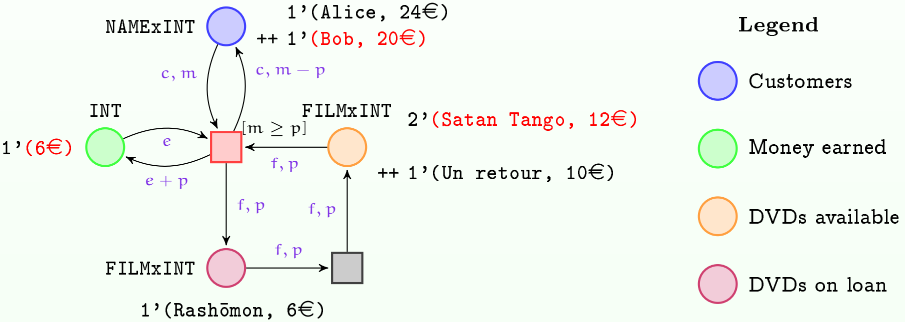

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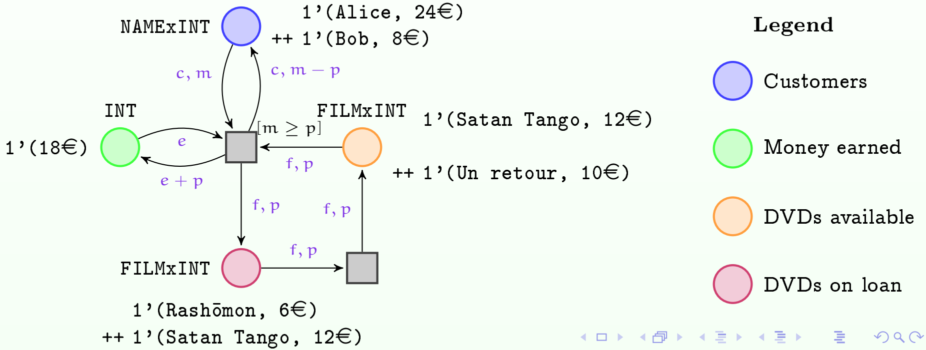

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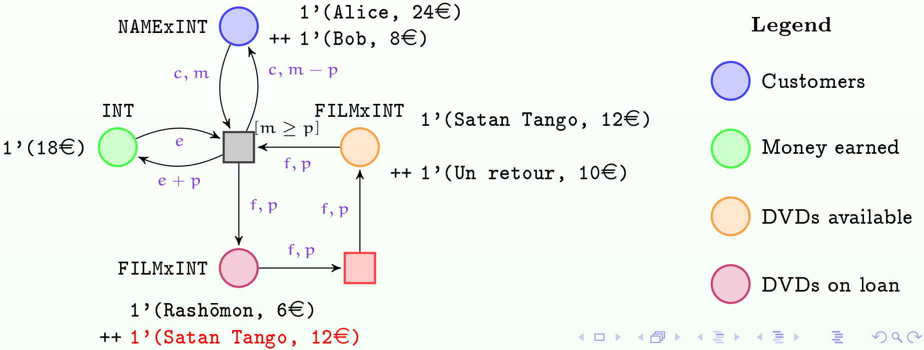

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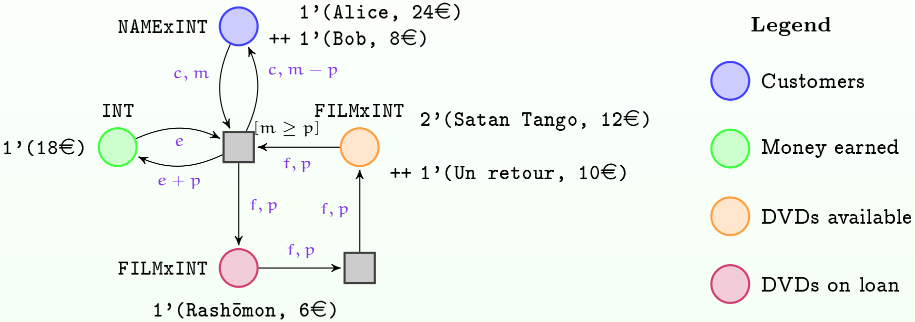

Perspectives (2/2)

- Termination
  - Probably does not terminate in the general case
  - . . . but no example exhibited so far

- Implementation
  - To do

- Modular analysis
  - Combine the inverse method with modular state space exploration for timed Petri nets [Lakos and Petrucci, 2007]
  - Idea: apply the inverse method to separate modules, then combine the result
  - Challenge: identify subclasses of time(d) Petri nets such that this applies

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The Algorithm

Algorithm 1: $IM(\mathcal{P}, \pi_0)$

$i \leftarrow 0$; $K \leftarrow K_{init}$; $C \leftarrow \{\mathbf{c}_0\}$
while `true` **do**
    while $\exists$ π_0-*incompatible classes in* C **do**
        Select a π_0-incompatible class (M, D) of C
        Select a π_0-incompatible J in $D{\downarrow_P}$
        $K \leftarrow K \wedge \neg J$
        $C \leftarrow \bigcup_{j=0}^{i} Post^{j}_{\mathcal{P}(K)}(\{\mathbf{c}_0\})$
    if $Post_{\mathcal{P}(K)}(C) \subseteq C$ **then**
        return $K_0 \leftarrow \bigcap_{(M,D) \in C} D{\downarrow_P}$
    $i \leftarrow i + 1$; $C \leftarrow C \cup Post_{\mathcal{P}(K)}(C)$

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