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A unified formalism for monoprocessor schedulability analysis under uncertainty

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Context: Verifying critical real-time systems

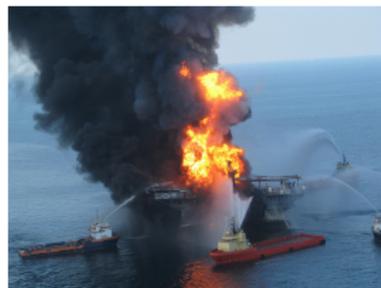
real-time systems:

- Systems for which not only the correctness but also the **timely** answer is important

Context: Verifying critical real-time systems

■ Critical real-time systems:

- Systems for which not only the correctness but also the **timely** answer is important
- Failures (in correctness or timing) may result in **dramatic** consequences



Real-time system

A real-time system is made of a set of **tasks** to execute on a **processor**

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- **W**: its worst-case execution time
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Tasks have **instances** that can be activated...

- periodically
- sporadically (usually with a minimum interarrival time)
- or following more complex patterns (e. g., activation following the completion of another task instance)

Scheduler

Activated instances are **queued**

When the processor is idle, which instance in the queue should be executed?

↪ decision made by the **scheduler**

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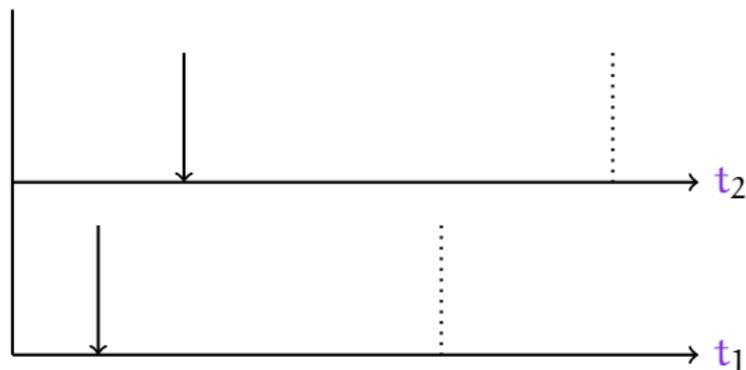
↪ decision made by the **scheduler**

The scheduler can be **preemptive**

- The execution of a lower priority task can be **interrupted** when a instance of a task with **higher priority** is activated
- After completion of the higher priority task, the lower priority task resumes

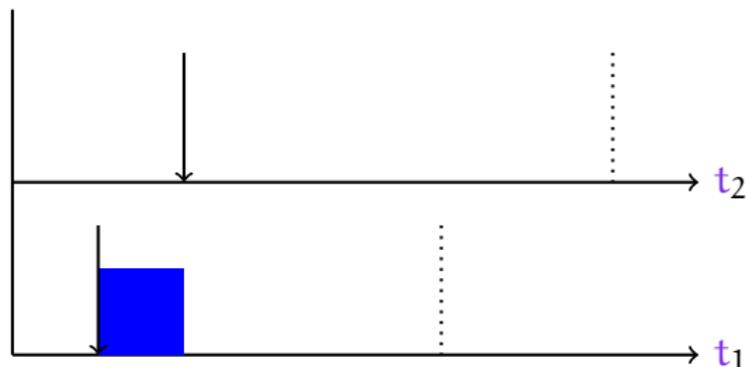
Example: preemptive fixed priority scheduler (FPS)

Task	B	W	D	priority
t_1	3	3	4	low
t_2	2	2	5	high



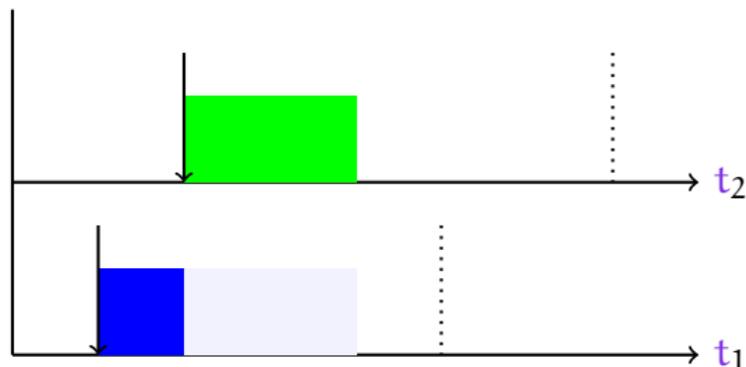
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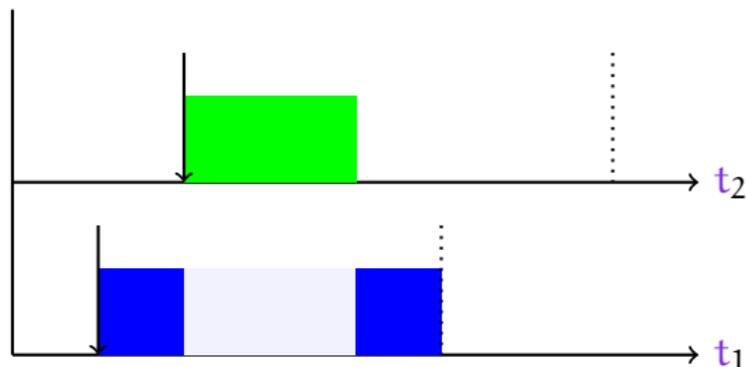
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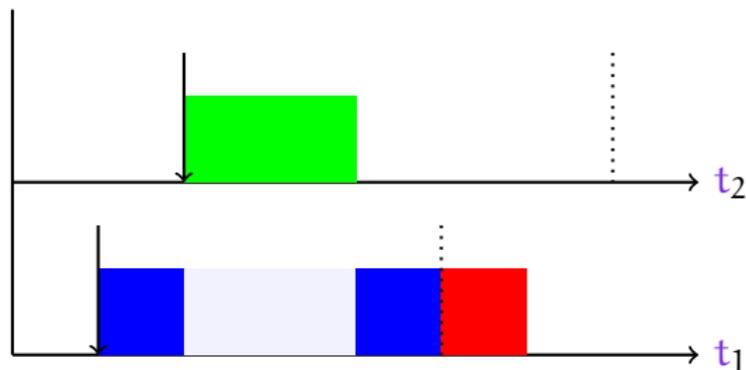
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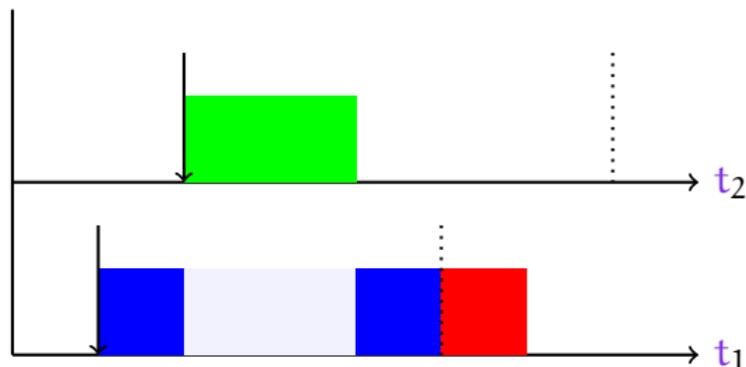
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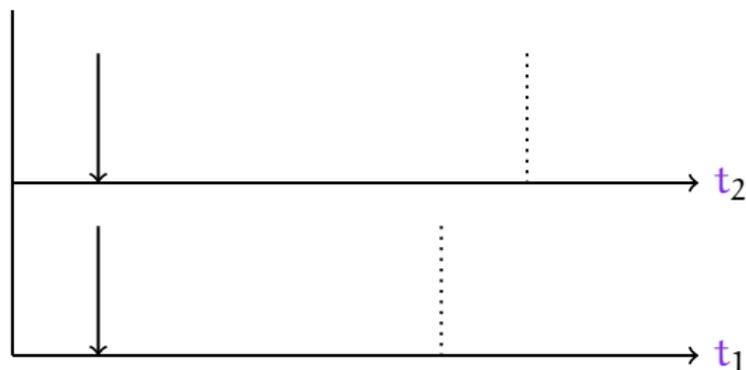
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Task t_1 misses its deadline

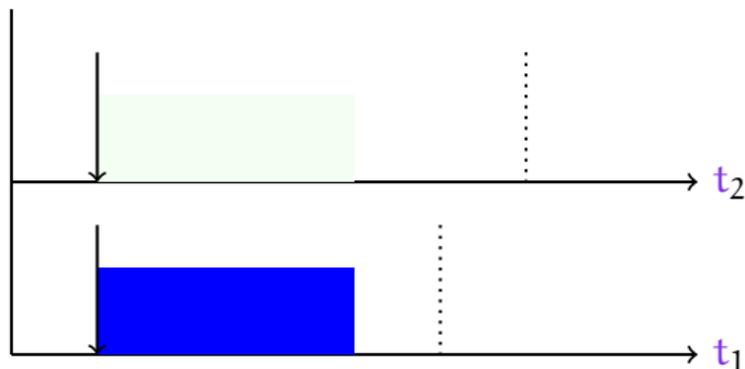
Example: earliest deadline first (EDF)

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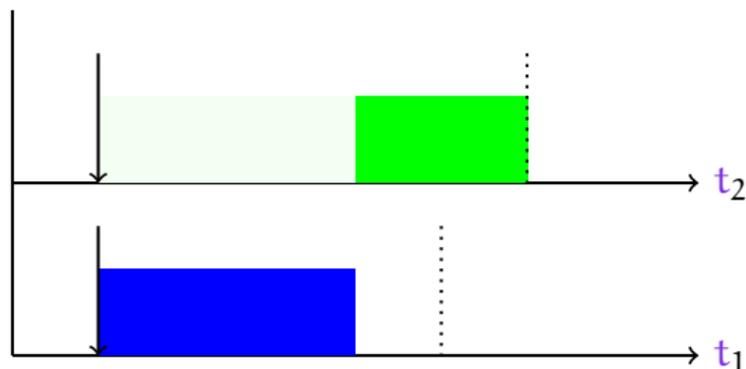
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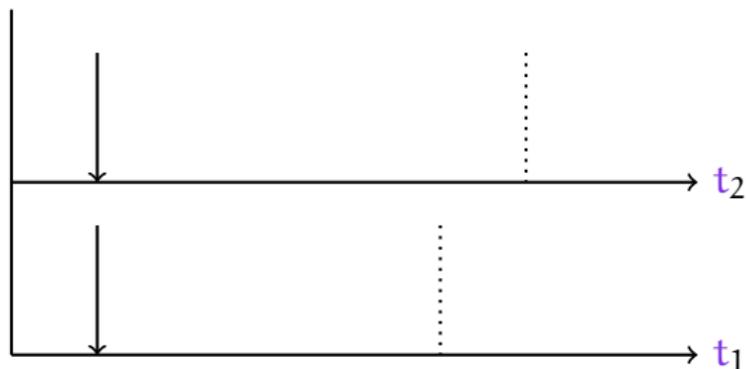
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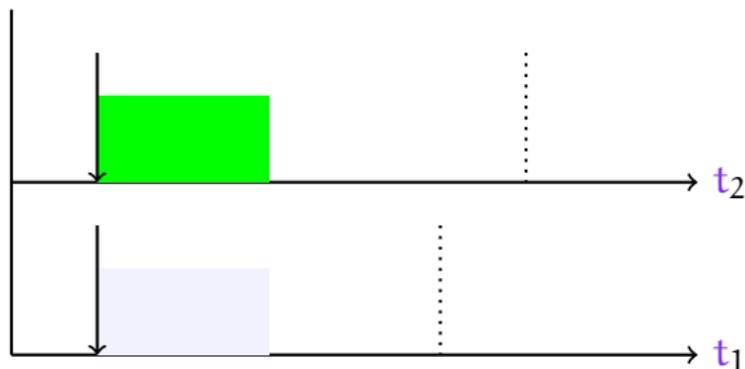
Example: shortest job first (SJF)

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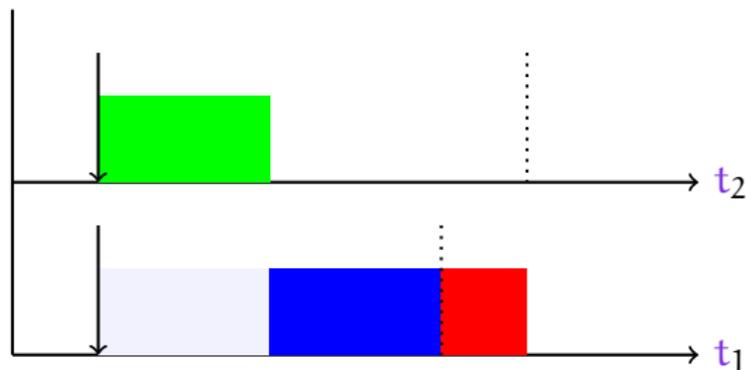
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Task t_1 misses its deadline

Schedulability analysis

Definition (schedulability analysis)

Given a real-time system and a scheduling policy, the schedulability analysis checks whether the system is schedulable (i. e., **all tasks meet their deadline**) *for all possible behaviors*.

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All possible behaviors:

- Depends on the periods, interarrival rates, dependencies between tasks...

Problem: schedulability analysis under uncertainty

Problem: what if some timing constants (deadlines, execution times, periods, interarrival times...) are **unknown** or **known with a limited precision**?

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Objective

Propose a framework for the monoprocessor schedulability analysis of real-time systems under uncertainty

Outline

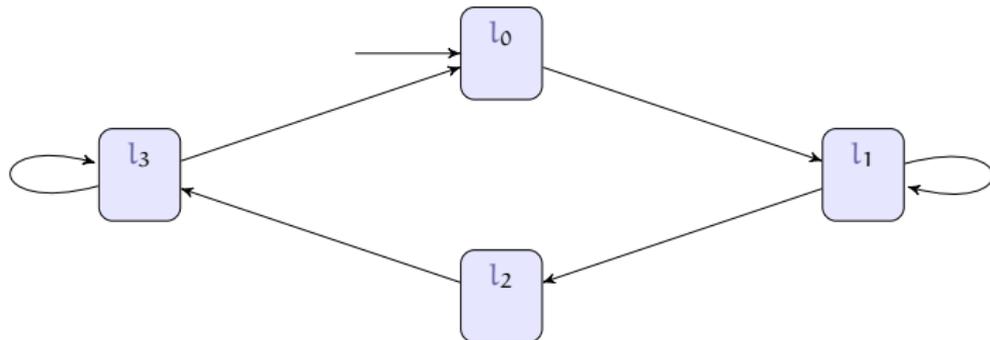
- 1 Parametric task automata
- 2 Decidability and undecidability
- 3 Schedulability under uncertainty
- 4 Conclusion and perspectives

Outline: Parametric task automata

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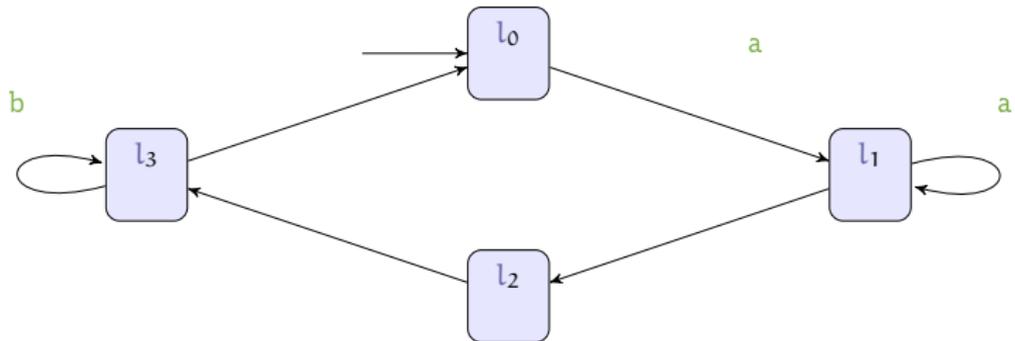
Task automaton (TaskA)

- Finite state automaton (sets of locations)



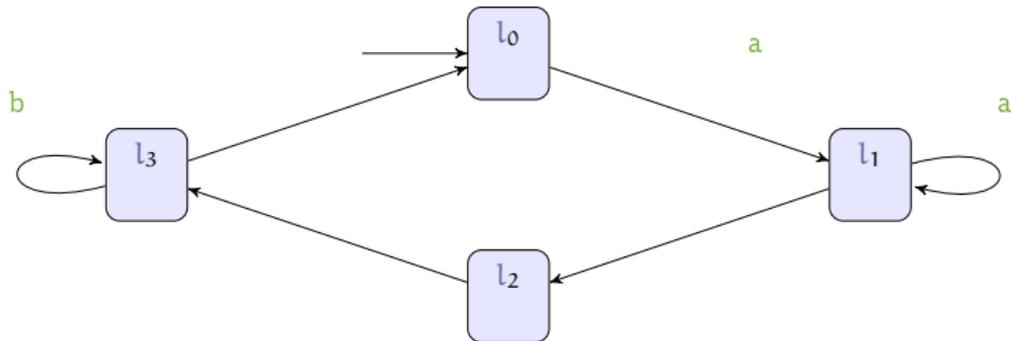
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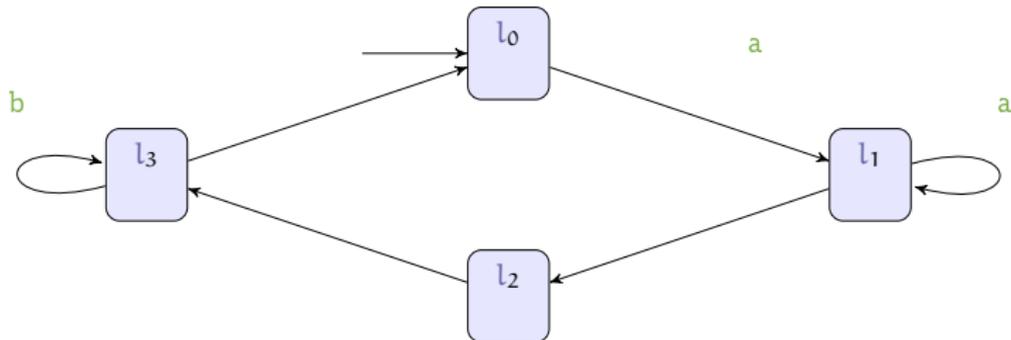
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- Finite state automaton (sets of **locations** and **actions**) with a set X of **clocks** as in timed automata [Alur and Dill, 1994]
 - Real-valued variables evolving linearly at the same rate



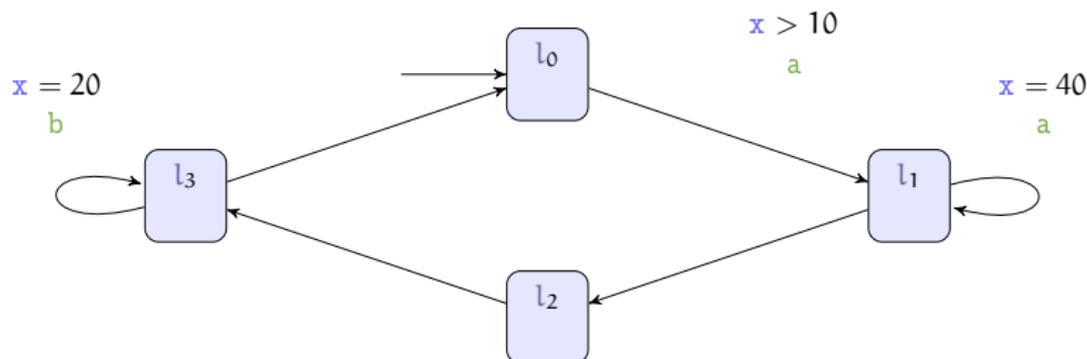
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 - Location **invariant**: property to be verified to stay at a location



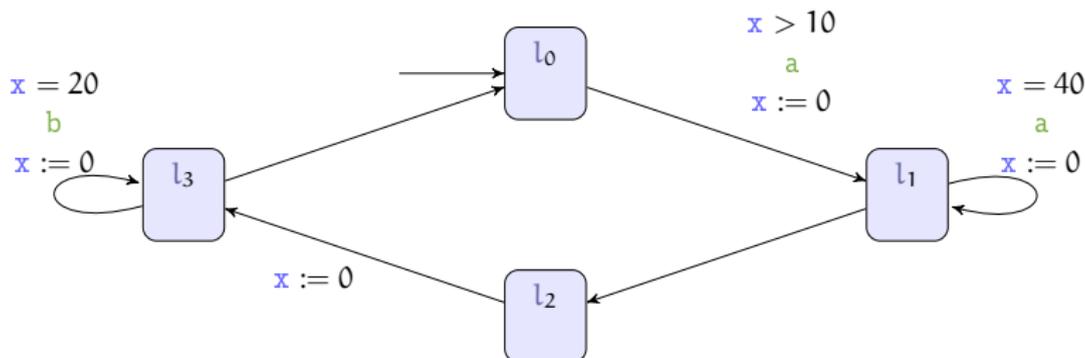
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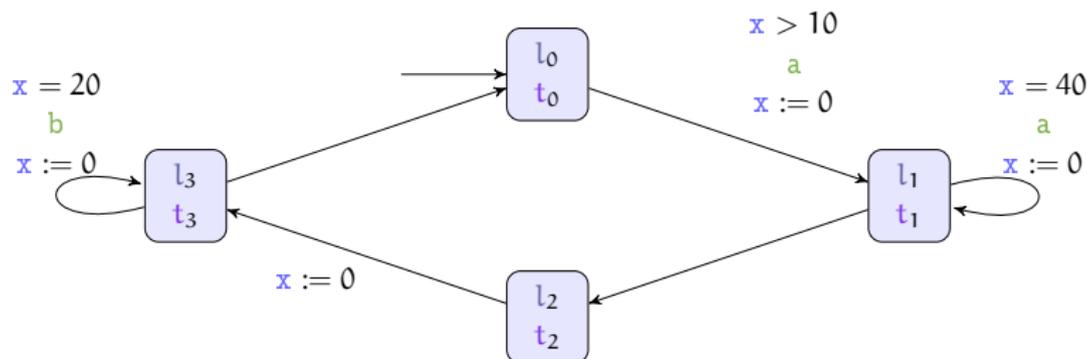
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Task automaton (TaskA)

- Finite state automaton (sets of **locations** and **actions**) with a set X of **clocks** as in timed automata [Alur and Dill, 1994]
 - Real-valued variables evolving linearly at the same rate
- a set \mathcal{T} of **tasks** [Norström et al., 1999, Fersman et al., 2007]
- Features
 - Location **invariant**: property to be verified to stay at a location
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Concrete semantics of task automata

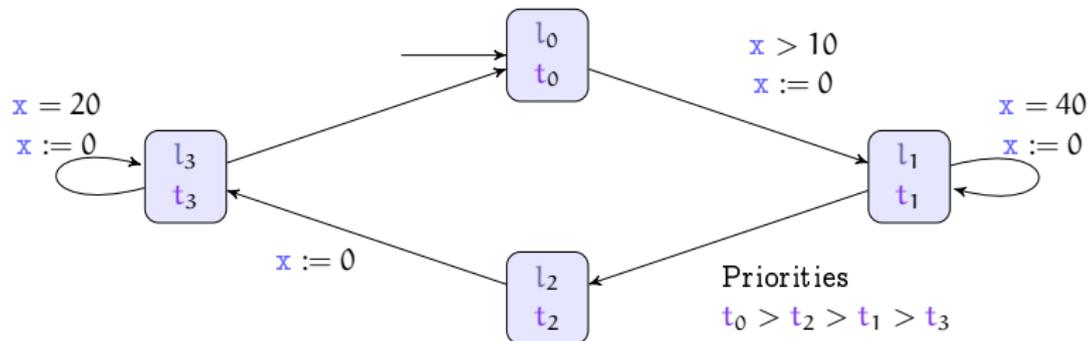
- **Concrete state** of a TaskA: triple (l, w, q) , where
 - l is a location,
 - w is a valuation of each clock
 - q is a task queue made of instances of \mathcal{T}
 - Instance: task, remaining BCET and WCET, remaining deadline

Example: $\left(l_1, \begin{pmatrix} x=1.2 \\ y=3.7 \end{pmatrix}, [(t_0, 0, 0.5, 1.5), (t_1, 4, 4, 18.5)(t_1, 4, 4, 19.5)] \right)$

- **Concrete run**: alternating sequence of concrete states and actions or time elapse according to a given scheduler

Parametric task automaton (PTaskA)

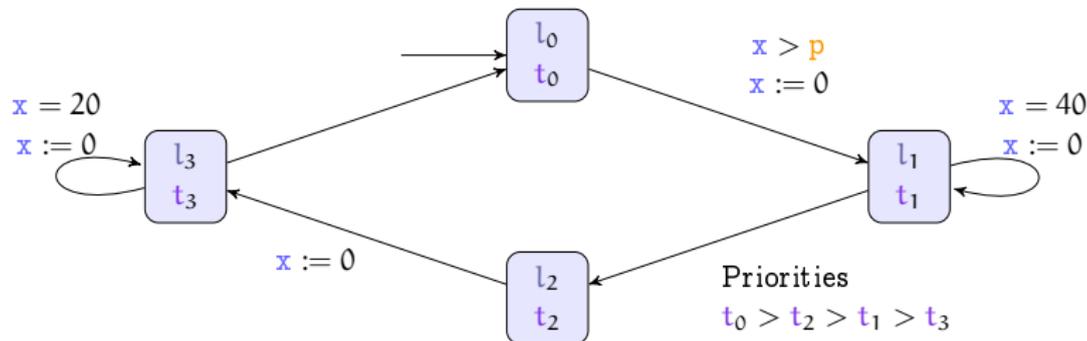
■ Task automaton



Task	B	W	D
t_0	0	1	2
t_1	4	4	20
t_2	0	1	4
t_3	2	2	10

Parametric task automaton (PTaskA)

- Task automaton extended with a set P of timing parameters, that can be used
 - in the automaton, and/or
 - in the tasks B , W and D



Task	B	W	D
t_0	0	1	2
t_1	4	4	20
t_2	0	1	p'
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Problems of interest

Parametric task automata can model real-time systems...

- that are periodic, sporadic, or correspond to more complex behaviors
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Some problems of interest

- For what values of the parameters is the system schedulable?
- Or even can we find at least one such valuation?
- Or is the system robustly schedulable? [Markey, 2011]

Outline: Decidability and undecidability

- 1 Parametric task automata
- 2 Decidability and undecidability**
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Decision problem

Schedulability-emptiness problem:

INPUT: A PTaskA \mathcal{A} and a scheduling strategy Sch

PROBLEM: is the set of valuations \mathbf{v} for which $\mathbf{v}(\mathcal{A})$ is schedulable for strategy Sch empty?

An undecidability result

Theorem (Undecidability)

*The schedulability-emptiness problem is **undecidable** for general $P\text{TaskA}$.*

An undecidability result

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*The schedulability-emptiness problem is **undecidable** for general P TaskA.*

Two reasons:

- Parametric task automata are at least as expressive as parametric timed automata [Alur et al., 1993] for which most non-trivial problems are undecidable [André, 2017]
- The schedulability of general non-parametric task automata is undecidable [Fersman et al., 2007] (in particular when using preemption)

Restricting a bit the formalism

Definition

A PTaskA has **schedulable-bounded parameters** if, for each task t , its worst-case execution time W is bounded in $[a, \infty)$ or $[a, b]$ with $a > 0$, and its deadline D is bounded in $[a, b]$, with $a, b \geq 0$.

Necessary to bound the task queue

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Definition

A PTaskA is an **L/U-PTaskA** if its parameters set is partitioned into lower-bound parameters (i. e., of the form $p < x$ or $p \leq x$) and upper-bound parameters (i. e., of the form $p > x$ or $p \geq x$).

Similar to L/U-parametric timed automata

[Hune et al., 2002]

A decidability result

Theorem (Decidability)

*The schedulability-emptiness problem is **decidable** for L/U -PTaskAs with schedulable-bounded parameters*

- 1 for non-preemptive FPS and SJF, and*
- 2 non-preemptive EDF without parametric deadlines.*

A decidability result

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- 1 for non-preemptive FPS and SJF, and*
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Proof idea

Reusing the encoding of [Norström et al., 1999, Fersman et al., 2007] together with a decidability result for L/U-parametric timed automata proved in [André, 2017]

Outline: Schedulability under uncertainty

- 1 Parametric task automata
- 2 Decidability and undecidability
- 3 Schedulability under uncertainty**
 - Parametric schedulability
 - Implementation in IMITATOR
 - Examples of analyses
- 4 Conclusion and perspectives

Parametric schedulability with parametric TaskA

Parametric schedulability reduces to **reachability synthesis**

- “Synthesize parameter valuations for which a deadline violation is **reachable**”
- Transform to parametric stopwatch automata [\[Sun et al., 2013\]](#)
 - The PTaskA itself can be seen as a parametric timed automaton
 - Any common scheduler can be transformed into a parametric stopwatch automaton
 - The system is made of the synchronous composition of both

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Undecidable in general... but we adopt a pragmatic approach and can use semi-algorithms or approximations

A practical encoding

In practice, we use extensions of parametric stopwatch automata

- Discrete global integer-valued variables (to model the queue)
- Extensive use of stopwatches
 - Also helps to reduce the state space!

Automatic translation of the scheduler into the IMITATOR input format

IMITATOR

Under continuous development since 2008

[André et al., 2012]

A library of benchmarks

- Communication protocols
- Schedulability problems
- Asynchronous circuits
- ... and more

Free and open source software: Available under the GNU-GPL license



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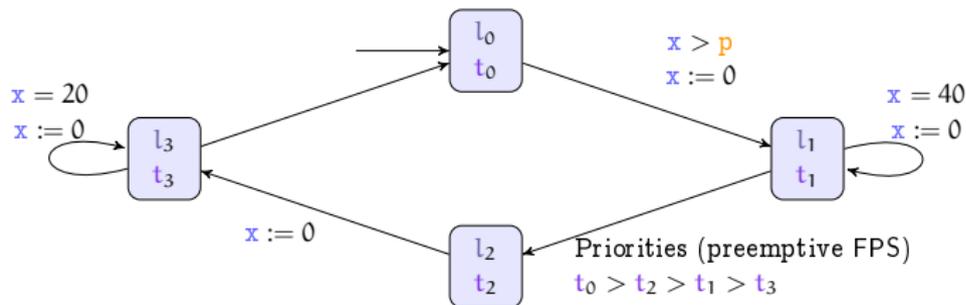
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Try it!

`www.imitator.fr`

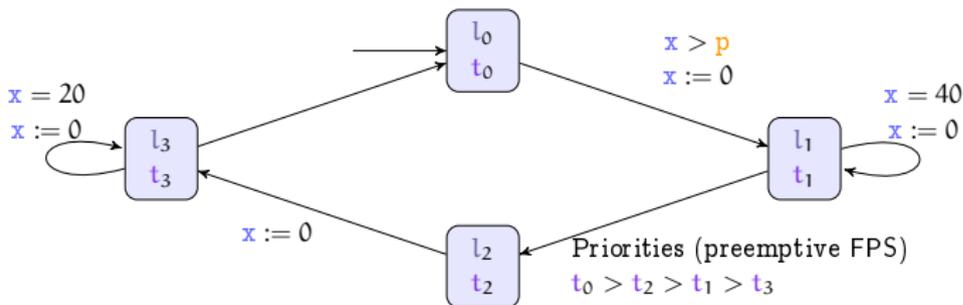
Parametric schedulability analysis



Task	B	W	D
t_0	0	1	2
t_1	4	4	20
t_2	0	1	p'
t_3	2	2	10

For which values of p and p' is the system schedulable?

Parametric schedulability analysis



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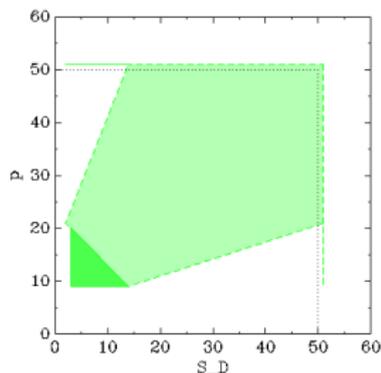
For which values of p and p' is the system schedulable?

$$p \geq 9 \wedge p' \geq 2 \wedge p + p' \geq 23$$

$$\vee$$

$$p \geq 9 \wedge p' \geq 3 \wedge p + p' < 23$$

FKPY07-fig2b-RSQP-generated



Robustness analysis

Definition

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Methodology:

- 1 Replace any guard $x \leq c$ with $x \leq c + \epsilon$
- 2 Replace any guard $x \geq c$ with $x \geq c - \epsilon$
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- 4 Check whether ϵ may be different from 0

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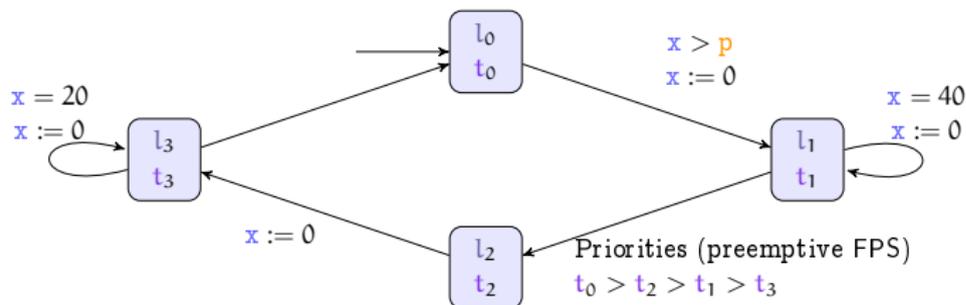
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For our TaskA (fixing $p = 10$ and $p' = 4$), we get $\epsilon = 0$

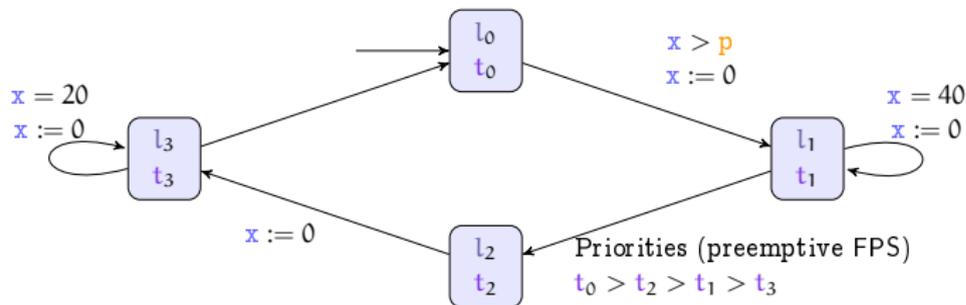
Parametric schedulability and robustness



Task	B	W	D
t_0	0	1	2
t_1	4	4	20
t_2	0	1	4
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For which values of p is the system robustly schedulable?

Parametric schedulability and robustness



Task	B	W	D
t_0	0	1	2
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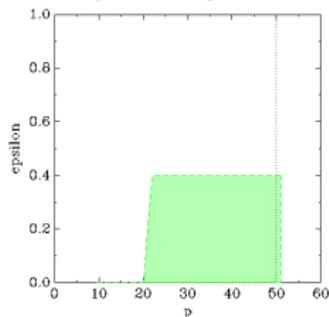
For which values of p is the system robustly schedulable?

$$p \geq 9 \wedge \epsilon = 0$$

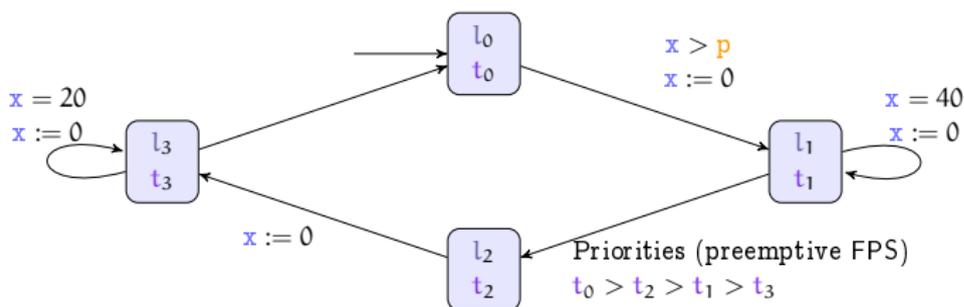
$$\vee$$

$$\epsilon \leq \frac{2}{5} \wedge p \geq 20 + 5\epsilon \wedge p \geq 19 + 8\epsilon$$

FKPY07-fig2b-RSQP-generated-robust



Parametric schedulability and robustness



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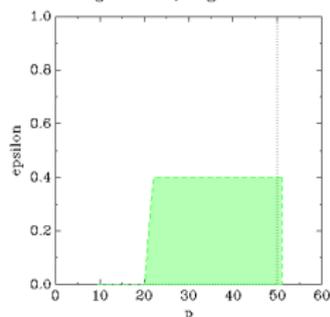
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$$\epsilon \leq \frac{2}{5} \wedge p \geq 20 + 5\epsilon \wedge p \geq 19 + 8\epsilon$$

FKPY07-fig2b-RSQP-generated-robust



We even know by how the system is robust (depending on p and ϵ)

Outline: Conclusion and perspectives

- 1 Parametric task automata
- 2 Decidability and undecidability
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Conclusion

Parametric task automata

- A unified, compact and expressive formalism to model and verify real-time systems under uncertainty
- ☹ Some undecidability results
- ☺ Some decidability results

Allow for

- Parametric schedulability
- Robustness analysis
- Robust parametric schedulability

Implementation in IMITATOR using an automated translation of the scheduler

Perspectives

Fill the decidability gap

- Still some unknown between decidability and undecidability results
- Our examples all fit into the undecidability cases... but analysis terminates with an exact answer

Design patterns for TaskA

- Allow to build periodic tasks, sporadic tasks... easily using predefined building blocks

Multiprocessor

- Extend the formalism to multiprocessor schedulability analysis

Mixed-criticality scheduling

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Additional explanation

Explanation for the 4 pictures in the beginning



Allusion to the Northeast blackout (USA, 2003)
 Computer bug
 Consequences: 11 fatalities, huge cost
 (Picture actually from the Sandy Hurricane, 2012)



Error screen on the earliest versions of Macintosh



Allusion to the sinking of the Sleipner A offshore platform (Norway, 1991)
 No fatalities
 Computer bug: inaccurate finite element analysis modeling
 (Picture actually from the Deepwater Horizon Offshore Drilling Platform)



Allusion to the MIM-104 Patriot Missile Failure (Iraq, 1991)
 28 fatalities, hundreds of injured
 Computer bug: software error (clock drift)
 (Picture of an actual MIM-104 Patriot Missile, though not the one of 1991)

Some success stories with IMITATOR

- Modeled and verified an **asynchronous memory circuit** by ST-Microelectronics
 - Project ANR Valmem
- Parametric schedulability analysis of a prospective architecture for the flight control system of the **next generation of spacecrafts** designed at ASTRIUM Space Transportation [Fribourg et al., 2012]
- Formal timing analysis of **music scores** [Fanchon and Jacquemard, 2013]
- Solution to a challenge related to a **distributed video processing system** by Thales

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