# Verification of an industrial asynchronous leader election algorithm using abstractions and parametric model checking

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#### **PLAN**

- 1. Motivation
- 2. Bully algorithm in the a/synchronous context
- 3. Adaptation of the Bully algorithm
- 4. Proofs
  - direct automated proof for a small number p of processes
  - proof with abstractions for <= 5000 processes</li>
- 5. Conclusion

## I. Motivation

#### Basic facts about leader election algorithms

- Many distributed algorithms needs one process to act as a leader or coordinator
- Does not matter which process does the job, just need to pick one
- Election algorithm technique to pick a unique coordinator
- Assumption: each process has a unique ID
   Goal: find the non-crashed process with highest ID
- Problem (Leader election): each node eventually decides whether it is leader or not, subject to the constraint that there is a unique leader
- Nodes are in one of the three states: leader, follower, candidate
- When leaving the candidate mode, a node goes into a final state
   (either leader or follower)

**II**.

Bully algorithm in the a/synchronous context

#### Bully algorithm in the a/synchronous setting

- Topology (here): complete graph
- Synchronous case:
  - All the process clocks are synchronized; processes update their state simultaneously
  - Bully algorithm [Garcia-Molina 1982]: classical synchronous leader election
- Asynchronous case:
  - every process is activated <u>periodically</u>, but <u>period</u> not (exactly) the same for each process (each period takes here its value in [49,51]).
  - besides, the value of each period may slowly evolves (jitter).
  - Initially, the values clocks are different (setoff).

# Short history of asynchronous versions of Bully algorithm

- [GM 1982] claims that the asynchronous version works (with correctness proof similar to the synchronous case).
- [Stoller 1997] gives a counterexample!
- [Svensson 2008] gives a corrected version, but:
  - the algorithm requires an important modification
  - hundreds of invariants (generated by hand) are needed for the semi-automated proof.

# III. A variant of Bully algorithm

#### General assumptions

- All the IDs of the nodes are different
- Each node has the ability to send messages to all the nodes,
   and can store messages received from other nodes
- Nodes are either in mode On or mode Off (failure)
- A node in mode *On* is in one of the states
  - Follower (the node is not competing to become leader)
  - Candidate (the mode is competing to become leader)
  - Leader (the mode has declared itself to be leader)
- Each transmitted message is of the form: (SenderID, state)
  where state is the state On/Off of the sending node

#### Updating algorithm (synchronous setting)

At each clock tick, every *On* process sends to all the other processes its ID number Each process compares the received ID numbers to its own ID number and updates it

```
foreach message \in allMessages do
\mathbf{if}\ message.SenderID > node_i.id\ \mathbf{then}
state_{next} \leftarrow \mathsf{Follower}
higherIDreceived \leftarrow \mathsf{true}
    if ¬ higherIDreceived then
```

#### Property P to be proven:

After a certain number of clean rounds (rounds with no crash and no recovery),

- the process On with the higher ID is Leader, and
- all the other *On* processes are **Follower** (no *On* process is **Candidate**)

#### Complications (asynchronous setting)

• If clock ticks are not synchronized, the messages are not emitted (and received) simultaneously

#### Complications due to asynchronous clocks

Table 2: Jitter values for Example 1

	$jitter^1$	$jitter^2$	$jitter^3$
Node 1	0.5	-0.5	0.5
Node 2	0	0.1	0
Node 3	0.1	0.3	0.5

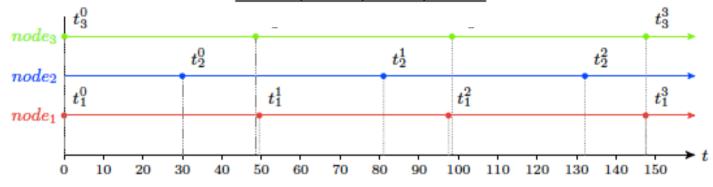


Fig. 1: Activation of three nodes with uncertain periods and jitters

- nb of activations for nodes 1 and 3 always the same up to a difference of 1 (due to the jitters) because they have same periods.
- But nb of activations for node 2 becomes smaller than that of nodes 1 and 3 by an increasing difference, since node 2 is slower (period: 51 instead of 49).
- This phenomenon does not occur when periods are equal for all nodes, and makes this setting more challenging.

#### A simple solution

 To overcome this difficulty, each ID proceeds to the update not at each period end, but every two (or more) periods

Basic insight:

**Lemma 1.** Assume a node i and activation times  $t_i^j$  and  $t_i^{j+2}$ . Then in between these two activations, node i received at least one message from all nodes.

#### Basic assumptions

- Instantiated model with uncertainty
  - Periods and jitters are known to belong to given intervals

•

Table 1: Constants (in ms)

Constant	Value
period <sub>mln</sub>	49
period <sub>max</sub>	51
jitter <sub>min</sub>	-0.5
jitter <sub>max</sub>	0.5

- the number p of processes is given
- The algorithm should work for p as large as possible

#### **Extended Bully algorithm**

#### Algorithm 1: UpdateNode(i)

```
1 if node: EvenActivation then
        allMessages \leftarrow ReadMailbox()
        higherIDreceived \leftarrow false
 3
        foreach message \in allMessages do
 4
            if message.SenderID > node_i.id then
 5
                 state_{next} \leftarrow Follower
 6
                higherIDreceived \leftarrow true
 7
       if ¬ higherIDreceived then
 8
            if node_i.state = Follower then
                 state_{next} \leftarrow Candidate
10
            else if node_i.state = Candidate then
11
                state_{next} \leftarrow Leader
12
            else if node_i.state = Leader then
13
                state_{next} \leftarrow Leader
14
       node_i.state \leftarrow state_{next}
16 node_i.EvenActivation \leftarrow \neg node_i.EvenActivation
17 message = \{node_i.id; node_i.state\}
18 Send_To_All_Network(message)
```

## Objective

- <u>Definition 1</u> (round). A *round* is a time period during which all the nodes that are *On* have sent at least one message.
- <u>Definition 2</u> (cleanness). A round is said to be <u>clean</u> if during its time period no node have been switched from <u>On</u> to <u>Off</u> or from <u>Off</u> to On.

The correctness property *P* that we want to prove automatically is:

« After 4 clean nodes, the node with the highest ID is recognized as the leader by all the *On* nodes of the network. »

### IV. PROOFS

#### IV.1 Direct proof of P using SMT solving

Using a model M of the algorithm, we get automatically a proof of P using SMT solver SafeProver [EJ17] when p is small (p <= 5).</li>

 This leads us to consider a method using abstractions to prove P for large values of p.

#### IV.2 Proof with abstractions

- we consider two abstractions of M
  - $1^{st}$  abstraction  $M^*$  consists in considering one of the p processes (arbitrarily), and consider the set of other processes under the form of a single big automaton (no timing information)
  - In the 2<sup>nd</sup> abstraction *T*, one considers two generic processes under the form of timed automaton with one parameter (the fixed value of the period lying in [49,51])

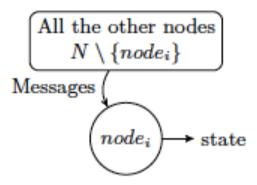
• we also **decompose** property *P* into several properties *P1-P2-P3-P4*.

## Scheme of the proof

For a given number p of processes, prove:

- P1-P2 on M\* with SMT solver (SafeProver)
- P3 on T with parametric timed model checker (IMITATOR)
   [NB: exact statement of P3 depends on values of periods and jitters]
- P4 on M\* with SMT solver using P1-P2-P3 as lemmas

# Automated proof of *P1-P2* for *M\** using SMT solver SafeProver



Scheme of model  $M^*$  with node i under study interacting with other nodes

```
 - P1: (Activation(j) ≥ 2 \land node_j.id ≠ maxId) ⇒ node_j.state = Follower 
 - P2: (Activation(j) ≥ 2 \land node_j.id = maxId) 
 ⇒ node_j.state ∈ {Candidate, Leader}
```

# Automated proof of *P3* for *T* using parametric timed model checker IMITATOR

Fig. 3: Component 1 of timed model T

For nodes  $node_i$  and  $node_j$ , the property that we want to specify corresponds in the direct model M (without abstraction) of Section 3 to:

$$- (Activation(i) \le 13 \land Activation(j) \le 13)$$
  
$$\Rightarrow |Activation(i) - Activation(j)| \le 2.$$

In our timed abstract model T, such a property becomes:

- (P3): 
$$\forall i \in \{1, ..., p\}$$
 Activation(j) ≤ 13 ⇒   
-2 ≤ Activation(j) - Activation(i) ≤ 1.

where Activation(i) denotes the number of activations of node i since the last clean round.

# Automated proof of *P4* for *M\** using SMT solver with *P1-P2-P3* as assumptions

 $P4: (Activation(i) \ge 4 \land node_i.id = maxId) \Rightarrow node_i.state = Leader$ 

#### Conclusion and final remarks

- We considered an asynchronous leader election algorithm
- We proved automatically its correctness property P using SMT solving for a small number p of nodes
- Using two abstractions and a decomposition of *P*, we verify the algorithm using SMT and parametric timed model checking for *p* up to 5000.
- The algorithm considered here is actually a variant of the original algorithm designed by THALES (not available for confidentiality reasons).
- The same kind of proof has been done for the original algorithm
- We are now considering to prove formally the correctness of the two abstractions

## THANKS!