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Precise Robustness Analysis of Time Petri Nets with Inhibitor Arcs

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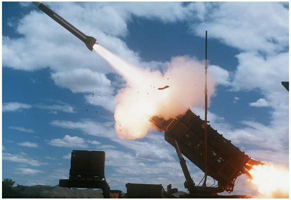
Context: Verifying Complex Timed Systems

- Need for early bug detection
  - Bugs discovered when final testing: expensive

⇝ Need for a thorough modeling and verification phase



Motivation: Robustness Analysis

- Timed systems are characterized by a set of timing constants
  - "The packet transmission lasts for $50\,ms$"
  - "The sensor reads the value every $10\,s$"
- Challenge: Robustness [Markey, 2011]
  - What happens if 50 is implemented with 49.99?
  - In which neighbourhood of 50 does the system behave well?

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- Timed systems are characterized by a set of timing constants
  - "The packet transmission lasts for $50\,ms$"
  - "The sensor reads the value every $10\,s$"
- Challenge: Robustness [Markey, 2011]
  - What happens if 50 is implemented with 49.99?
  - In which neighbourhood of 50 does the system behave well?
- Parametric analysis
  - Consider that timing constants are parameters
  - Find good values for the parameters, such that the system still behaves well

Outline

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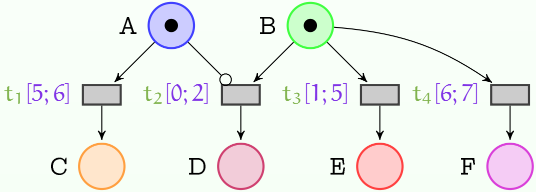
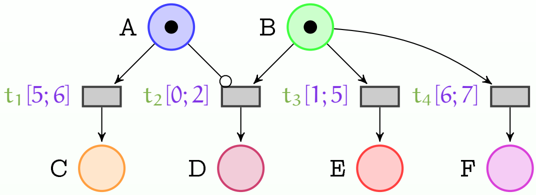
Time Petri Nets With Inhibitor Arcs (ITPNs)

- Powerful formalism for verifying real-time systems [Merlin, 1974]
- Transition t_1 can fire from 5 to 6 units of time after being enabled
- An enabled transition must fire before (or at) its upper bound
- An inhibitor arc enables its transition (t_2) when its source place (A) is empty



Time Petri Nets With Inhibitor Arcs: Example



Some possible runs

AB

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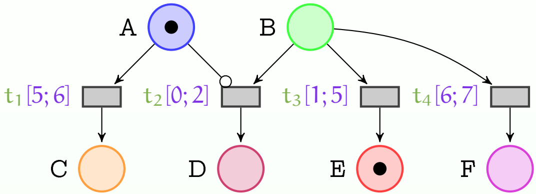

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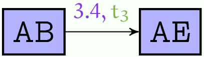

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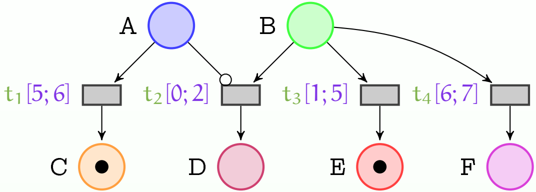

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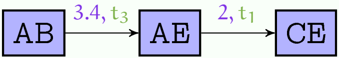

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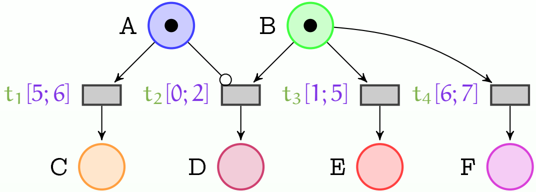

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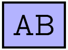

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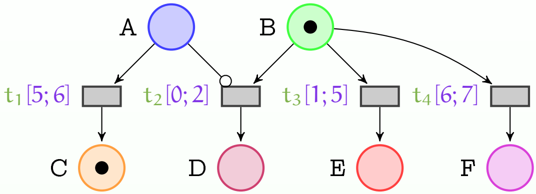

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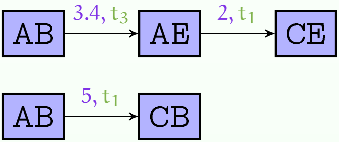

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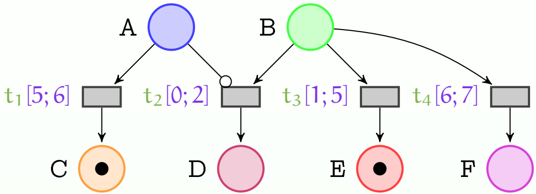

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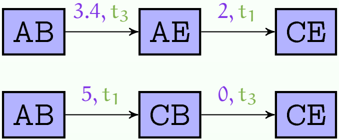

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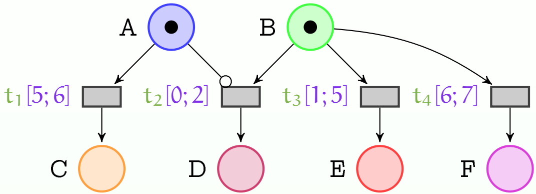


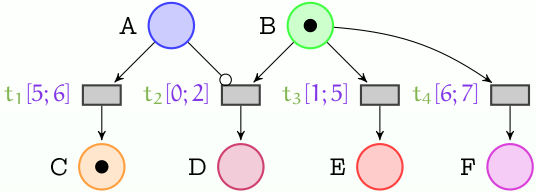
Some possible runs

AB $\xrightarrow{3.4,\ t_3}$ AE $\xrightarrow{2,\ t_1}$ CE

AB $\xrightarrow{5,\ t_1}$ CB $\xrightarrow{0,\ t_3}$ CE

AB

Time Petri Nets With Inhibitor Arcs: Example



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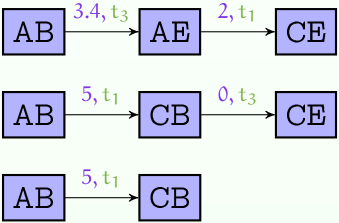

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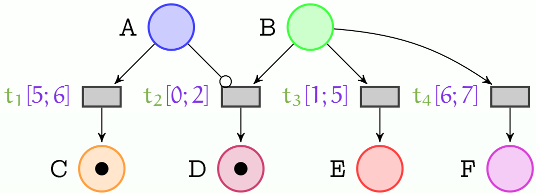

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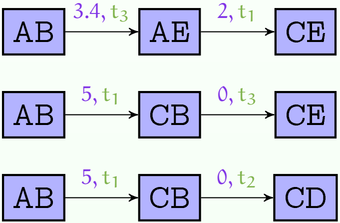

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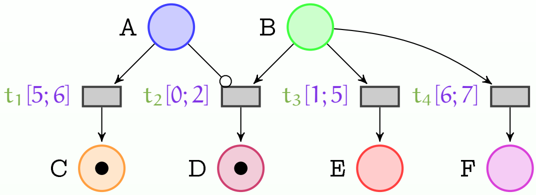

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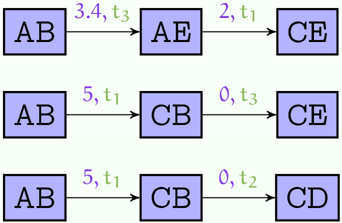
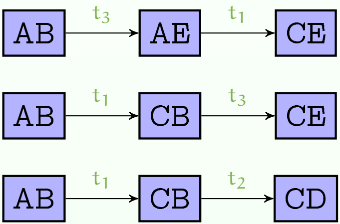
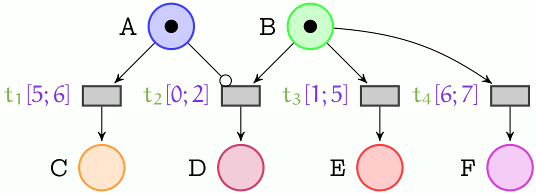


Trace set



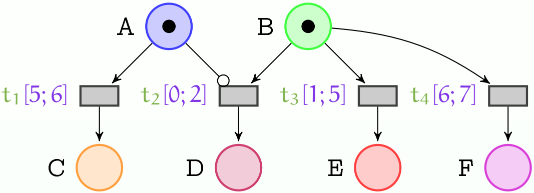
Trace: time-abstract behaviour

Robustness (1/2)



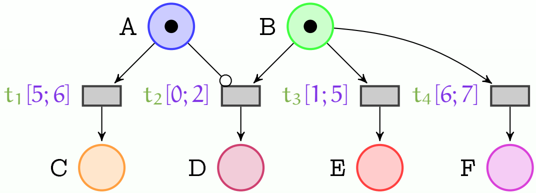
- What happens if $t_2[0;2]$ is implemented with $t_2[0.01;2]$?
  - Trace AB $\xrightarrow{t_1}$ CB $\xrightarrow{t_2}$ CD cannot happen anymore

Robustness (1/2)



- What happens if $t_2[0;2]$ is implemented with $t_2[0.01;2]$?
  - Trace $AB \xrightarrow{t_1} CB \xrightarrow{t_2} CD$ cannot happen anymore
- What happens if $t_3[1;5]$ is implemented with $t_3[1;4.99]$?
  - Trace $AB \xrightarrow{t_1} CB \xrightarrow{t_3} CE$ cannot happen anymore

Robustness (1/2)



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- What happens if $t_3[1;5]$ is implemented with $t_3[1;4.99]$?
  - Trace AB $\xrightarrow{t_1}$ CB $\xrightarrow{t_3}$ CE cannot happen anymore

$\rightsquigarrow$ This system is not robust, in the sense that infinitesimal variations of the bounds lead to a different discrete behaviour (trace set).

Robustness (2/2)

Definition (LT-robustness)

An ITPN N is LT-robust if there exists $\gamma > 0$ such that N_γ and N have the same trace sets.
(where N_γ is any ITPN similar to N where each timing bound c can be replaced with any value within $[c - \gamma, c + \gamma]$)

Challenges:

- Is an ITPN robust?
- If not, why is it non-robust?
- Is it possible to render robust a non-robust ITPN? If so, how?

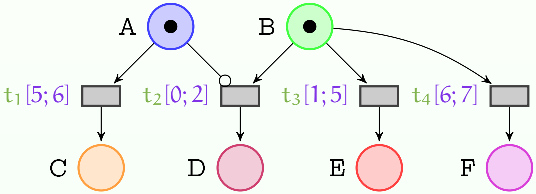
Outline

Parametric Time Petri Nets

Idea: Reason parametrically (using unknown constants)

Parametric Time Petri Nets with Inhibitor Arcs (PITPNs)

- Constants in firing intervals replaced with parameters [Traonouez et al., 2009]



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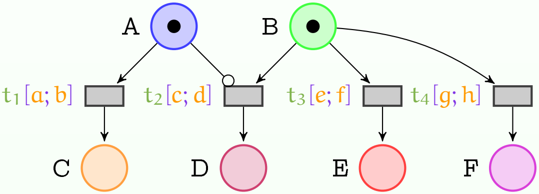

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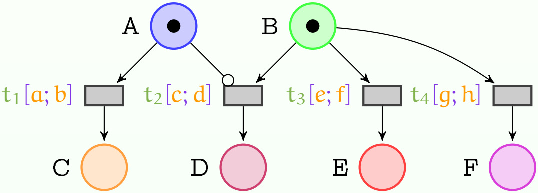
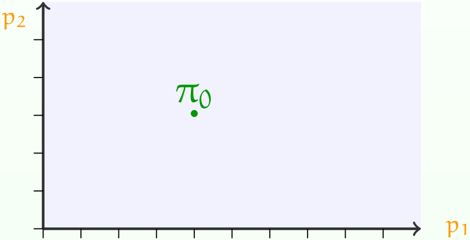


- Notation: given a PITPN $\mathcal{N}$ and a valuation π of the parameters, we denote by $[\![\mathcal{N}]\!]_\pi$ the ITPN obtained from $\mathcal{N}$ by replacing all parameters with their valuation in π

The Inverse Method (IM)

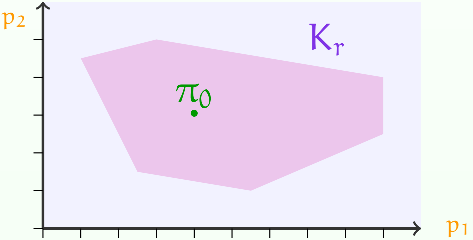
- Input
  - A PITPN $\mathcal{N}$
  - A reference valuation π_0 of all the parameters of $\mathcal{N}$



The Inverse Method (*IM*)

- Input
  - A PITPN $\mathcal{N}$
  - A reference valuation π_0 of all the parameters of $\mathcal{N}$

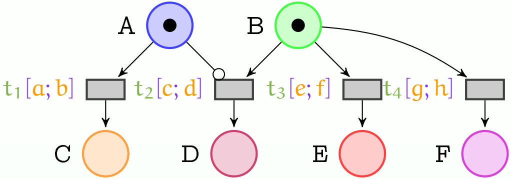
- Output: K_r
  - Convex constraint on the parameters such that
    - $\pi_0 \models K_r$
    - For all points $\pi \models K_r$, $[\![\mathcal{N}]\!]_\pi$ and $[\![\mathcal{N}]\!]_{\pi_0}$ have the same trace sets



The Inverse Method: General Idea

- Initially defined for timed automata [A., Chatain, Encrenaz, Fribourg, 2009]
- Extended to PITPNs [A., Pellegrino, Petrucci, 2013]
- The idea
  - Exploration of the parametric state space
  - Instead of negating bad states (as in "CEGAR" approaches), remove π_0-incompatible states
  - Return the intersection of all constraints on the parameters

Application to an Example

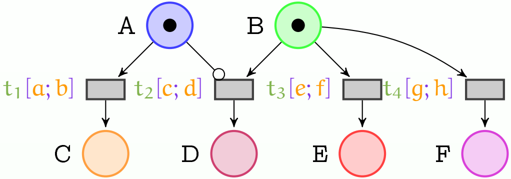


π_0

$a = 5$	$b = 6$
$c = 0$	$d = 2$
$e = 1$	$f = 5$
$g = 6$	$h = 7$

Forward analysis

Application to an Example



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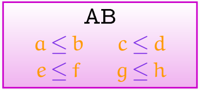
Forward analysis

K :

true



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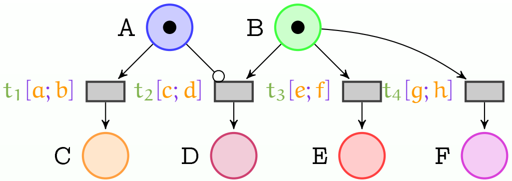

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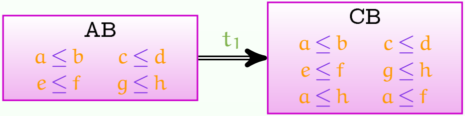

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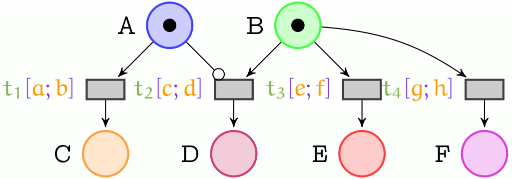

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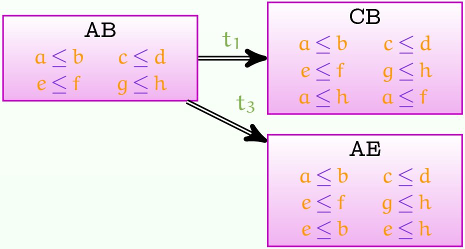

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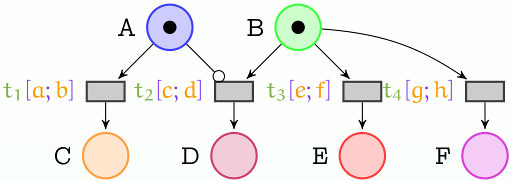

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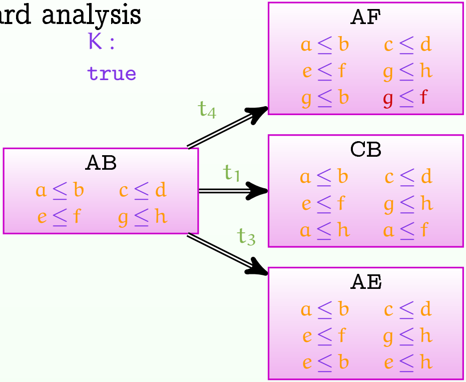

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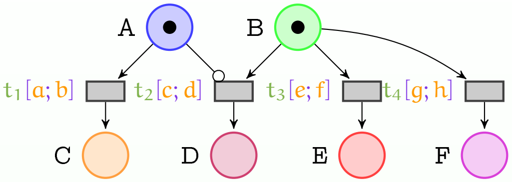

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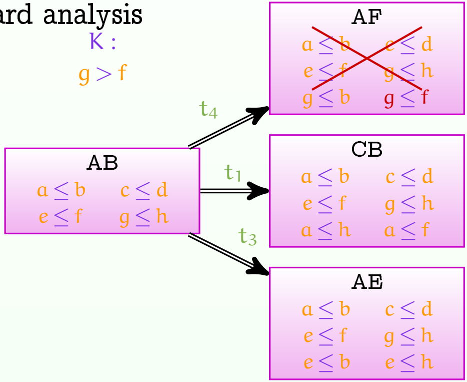
$g > f$



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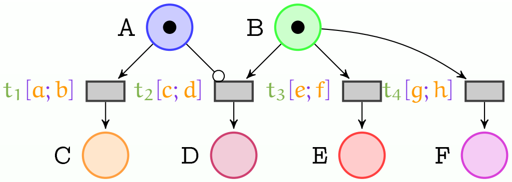

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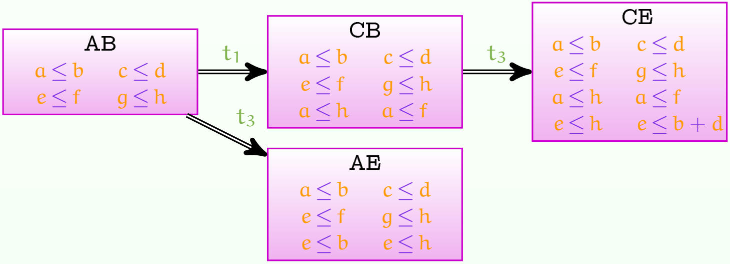

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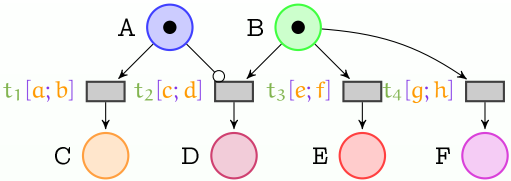

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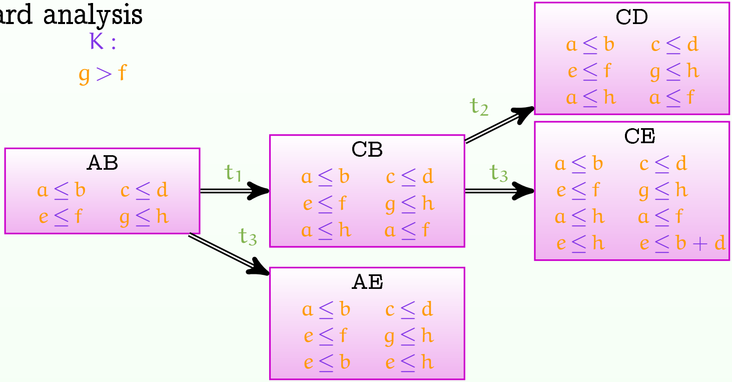

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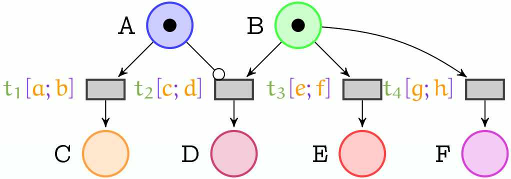

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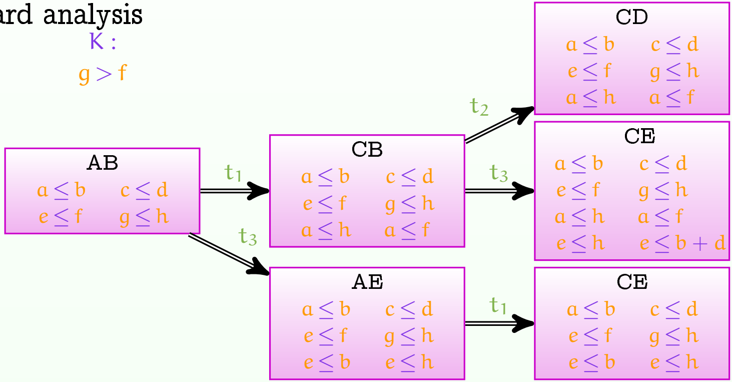

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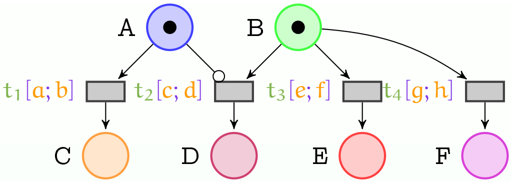

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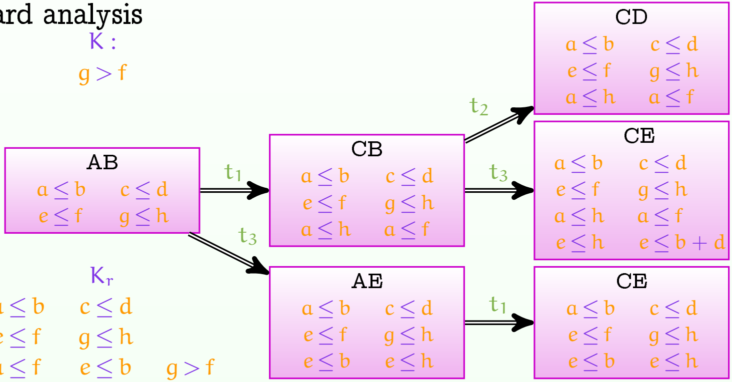


K_r

$a \leq b$ $c \leq d$
$e \leq f$ $g \leq h$
$a \leq f$ $e \leq b$ $g > f$

Properties

- Correctness
  - $\pi_0 \models K_r$ and
  - $\forall \pi \models K_r$, $[\![\mathcal{N}]\!]_\pi$ and $[\![\mathcal{N}]\!]_{\pi_0}$ have the same trace set.
- *IM* is non-confluent
  - Several executions with the same input may lead to different outputs
- *IM* is non-complete
  - K_r may not be the maximum set of parameter valuations with the same trace set as $[\![\mathcal{N}]\!]_{\pi_0}$
- Termination of *IM* is not guaranteed in general
  - Parameter synthesis for PITPNs undecidable [Traonouez et al., 2009]

Outline

Robustness Using the Inverse Method

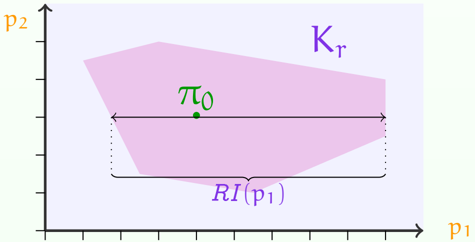
Let N be an ITPN.

General idea

1. Construct the parametric version $\mathcal{N}$ of N, and π_0 the reference valuation such that $[\![\mathcal{N}]\!]_{\pi_0} = \mathsf{N}$
2. Call $IM(\mathcal{N}, \pi_0)$ and assume K_r is the resulting constraint
3. Measure the system robustness
4. If the system is non-robust, render it robust (if possible)

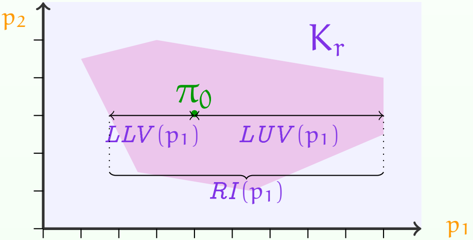
Metrics for Measuring Local Robustness

- Ranging interval of a parameter $RI(p)$
  - Minimum and maximum admissible values within K_r



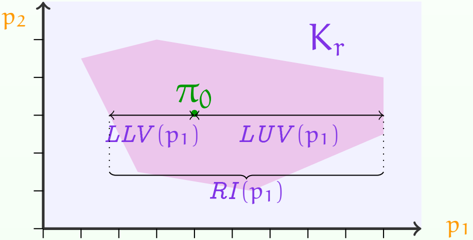
Metrics for Measuring Local Robustness

- Ranging interval of a parameter $RI(p)$
  - Minimum and maximum admissible values within K_r
- Local lower/upper variability of a parameter
  - Distance between $\pi_0(p)$ and and the lower/upper bound of $RI(p)$
  - Given $RI(p) = (a, b)$, then $LLV(p) = \pi_0(p) - a$ and $LUV(p) = b - \pi_0(p)$



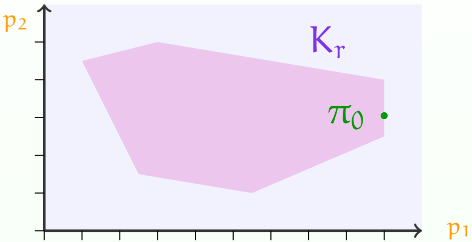
Metrics for Measuring Local Robustness

- Ranging interval of a parameter $RI(p)$
  - Minimum and maximum admissible values within K_r
- Local lower/upper variability of a parameter
  - Distance between $\pi_0(p)$ and and the lower/upper bound of $RI(p)$
  - Given $RI(p) = (a, b)$, then $LLV(p) = \pi_0(p) - a$ and $LUV(p) = b - \pi_0(p)$
- Local robustness: distance between $\pi_0(p)$ and the closest border within K_r
  - $LR(p) = \min(LLV(p), LUV(p))$



Critical Timing Bounds

- Critical timing bounds are those with a null local robustness



Remark

If any of the timing bounds is critical, classical ("Δ-based") approaches will just classify the system as non-robust.

Relaxing Timing Bounds

Definition (Potential robustness)

An ITPN N is potentially robust if, for all timing bounds p_i, $LLV(p_i) > 0$ or $LUV(p_i) > 0$.

Intuitively: A system is potentially robust if each parameter can vary within K_r.

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Theorem (Rendering a system robust)

If N is potentially robust, then there exists π_R such that $[\![\mathcal{N}]\!]_{\pi_R}$ is LT-robust, and has the same trace set as N.

Construction: choose $\frac{LLV(p)+LUV(p)}{2}$ for each parameter p.

Relaxing Timing Bounds: Remarks

The potential robustness is a non-necessary condition to render a system robust

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Relaxing Timing Bounds: Remarks

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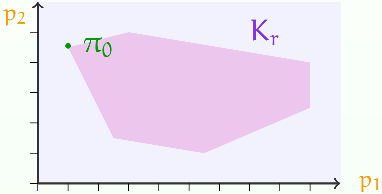
1. The potential robustness is based on the local robustness, that comes from K_r, that may be non-complete
2. The potential robustness considers the variability of each timing bound in an independent manner



- In that case, the system is not potentially robust (since $LLV(p_2) = LUV(p_2) = 0$), but could still be made robust (by choosing a point in the middle of K_r)

Comparison with Related Work (1/2)

- Robustness studied for timed automata and time Petri nets (see [Markey, 2011] for a survey)
- "Δ-based" approaches
  - Robustness studied with respect to a single enlargement Δ for all bounds
  - or to a single shrinking Δ for all bounds
  - Extension to a (constant) vector
  - Extension to independent variations Δ for each parameter, but for shrinking only [Sankur, 2013]

Comparison with Related Work (2/2)

- Recent approaches
  - Parameterized robust reachability in timed automata is decidable [Bouyer et al., 2012]
  - Computing the greatest acceptable variation Δ is decidable for flat timed automata with progressive clocks [Jaubert and Reynier, 2011]
  - CEGAR-based approach using parametric techniques to evaluate the greatest acceptable variation Δ for parametric timed automata (not decidable in general) [Traonouez, 2012]

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- In contrast to most approaches, we consider a local robustness measure for each delay
  - ☺ For linear-time properties
  - ☺ More flexible: Bounds can be both enlarged and shrinked
  - ☺ More precise: Exhibits the critical timing bounds
  - ☹ May not terminate

Outline

Conclusion

- Local robustness analysis of timed systems
  - For linear-time properties
  - Using the inverse method
  - Quantifies the robustness of each timing bound

    $\rightsquigarrow$ Identifies critical bounds
- Sufficient condition for rendering a non-robust system robust
- Comparison with related approaches
  - ☺ More precise than most existing approaches
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Conclusion

- Local robustness analysis of timed systems
  - For linear-time properties
  - Using the inverse method
  - Quantifies the robustness of each timing bound

  $\rightsquigarrow$ Identifies critical bounds

- Sufficient condition for rendering a non-robust system robust

- Comparison with related approaches
  - ☺ More precise than most existing approaches
  - ☹ May not terminate

- Linear-time properties, hence untimed
  - But timed properties can be considered using observers

Perspectives

- Implementation
  - Work in progress
  - Comparison with other tools such as Shrinktech [Sankur, 2013]
- Improve conditions for rendering non-robust systems robust
- Variation of the clocks speed ("ϵ")
  - Addition of two parameters for the admissible decrease and increase of the clock rate
  - Extension of the inverse method to non-linear (hybrid) systems

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Additional explanation

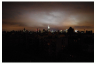
The Algorithm

Algorithm 1: *IM*$(\mathcal{N}, \pi)$

input : PITPN $\mathcal{N}$ of initial class c_0 and initial constraint K_0, valuation π
output: Constraint K_r

$i \leftarrow 0$; $K_c \leftarrow K_0$; $C \leftarrow \{c_0\}$
while true **do**
  while $\exists$ π-*incompatible classes in* C **do**
    Select a π-incompatible class (M, D) of C
    Select a π-incompatible J in $D{\downarrow_P}$
    $K_c \leftarrow K_c \wedge \neg J$; $C \leftarrow \bigcup_{j=0}^{i} Post^{j}_{\mathcal{N}(K_c)}(\{c_0\})$
  if $Post_{\mathcal{N}(K_c)}(C) \subseteq C$ **then**
    return $K_r \leftarrow \bigcap_{(M,D) \in C} D{\downarrow_P}$
  $i \leftarrow i + 1$; $C \leftarrow C \cup Post_{\mathcal{N}(K_c)}(C)$

Explanation for the 4 pictures in the beginning



Allusion to the Northeast blackout (USA, 2003)
Computer bug
Consequences: 11 fatalities, huge cost
(Picture actually from the Sandy Hurricane, 2012)

Allusion to any plane crash
(Picture actually from the happy-ending US Airways Flight 1549, 2009)

Allusion to the sinking of the Sleipner A offshore platform (Norway, 1991)
No fatalities
Computer bug: inaccurate finite element analysis modeling
(Picture actually from the Deepwater Horizon Offshore Drilling Platform)

Allusion to the MIM-104 Patriot Missile Failure (Iraq, 1991)
28 fatalities, hundreds of injured
Computer bug: software error (clock drift)
(Picture of an actual MIM-104 Patriot Missile, though not the one of 1991)

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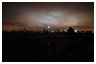
Source of the graphics used



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