

ATVA 2013

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Hà Nội, Việt Nam

Merge and Conquer

State Merging in Parametric Timed Automata

Étienne André, Laurent Fribourg, Romain Soulat

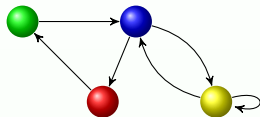
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Université Paris 13, Sorbonne Paris Cité, France



Context: Verifying Complex Timed Systems

- Use formal methods



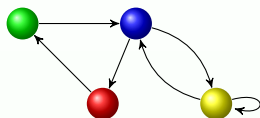
A **model** of the system

● is unreachable

A **property** to be satisfied

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$\models$

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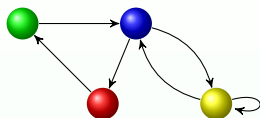
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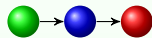
A **property** to be satisfied

- Question: does the model of the system **satisfy** the property?

Yes



No



Counterexample

Context: Parameter Synthesis

- Timed systems are characterized by a **set of timing constants**
 - “The packet transmission lasts for **50 ms**”
 - “The sensor reads the value every **10 s**”
- Verification for **one** set of constants does not usually guarantee the correctness for other values
- Challenges
 - **Numerous verifications**: is the system correct for any value within $[40; 60]$?
 - **Optimization**: until what value can we increase **10**?
 - **Robustness**: What happens if **50** is implemented with **49.99**?

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- **Parameter synthesis**
 - Consider that timing constants are unknown constants (**parameters**)
 - Find **good values** for the parameters

Outline

- 1 Parametric Timed Automata
- 2 Merging States in Timed Automata
- 3 Merging States in Parametric Timed Automata
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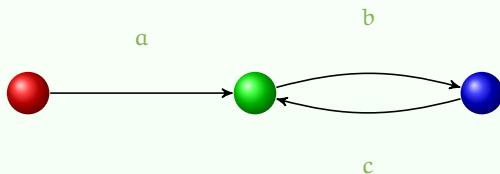
Timed Automaton

- Finite state automaton (sets of locations)



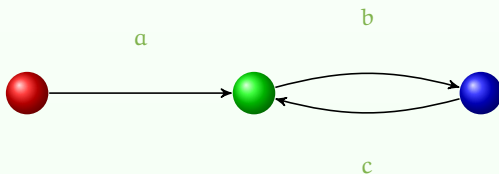
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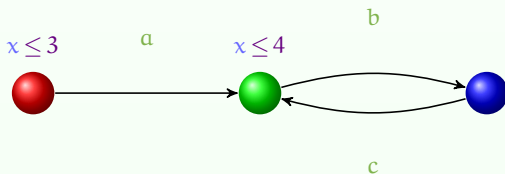
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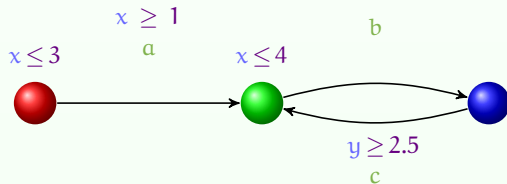
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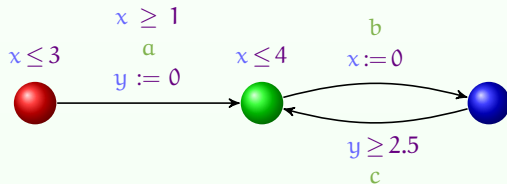
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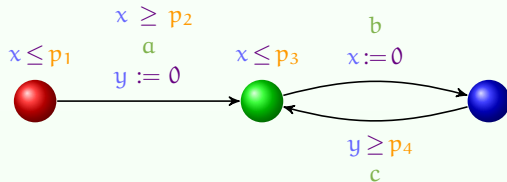
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 - Clock **reset**: some of the clocks can be set to 0 at each transition



Parametric Timed Automaton (PTA)

- Finite state automaton (sets of **locations** and **actions**) augmented with
 - A set X of **clocks** (i.e., real-valued variables evolving linearly at the same rate [Alur and Dill, 1994])
 - A set P of **parameters** (i.e., **unknown constants**), used in guards and invariants [Alur et al., 1993b]
- Features
 - Location **invariant**: property to be verified to stay at a location
 - Transition **guard**: property to be verified to enable a transition
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Semantics of Parametric Timed Automata

- **State** of a PTA: pair (l, C) , where
 - l is a **location** (e.g., ) ,
 - C is a **constraint** (conjunction of inequalities) over X and P

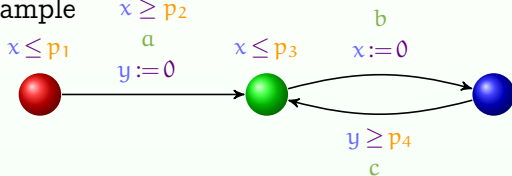
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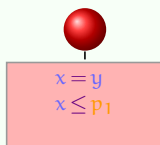
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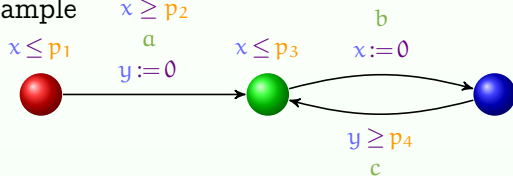
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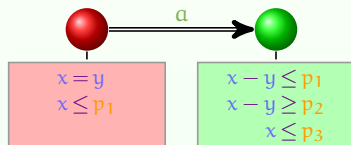
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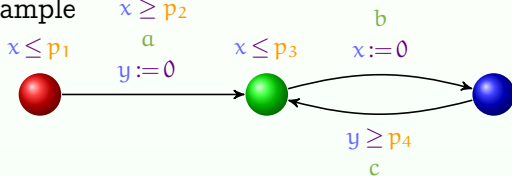
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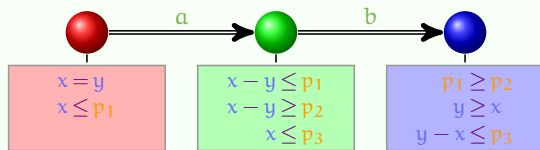
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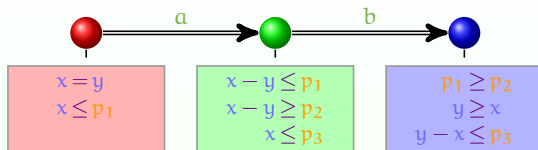


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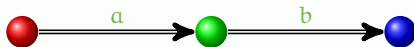
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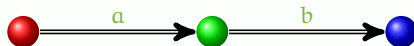
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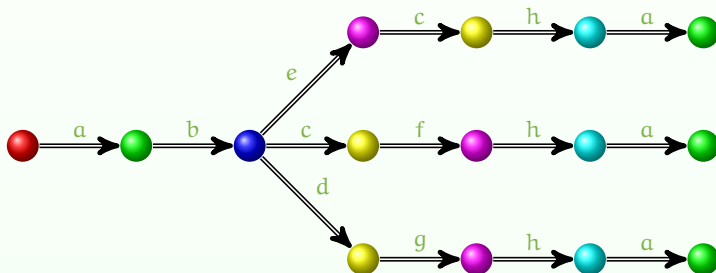
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- **Trace set** of a PTA: set of all its traces



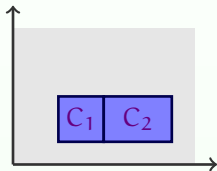
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State Merging

- In timed automata, two states $(\bullet, C_1)$ and $(\bullet, C_2)$ are **mergeable** if
 - 1 Their discrete part is equal ($\bullet = \bullet$), and
 - 2 Their union $C_1 \cup C_2$ is **convex**

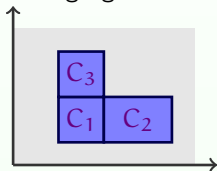
Their **merging** is $(\bullet, C_1 \cup C_2)$ [David, 2005, David, 2006]



- Preserves safety properties

State Merging: Implementation

- States in timed automata are often encoded using difference bound matrices (DBMs)
- The mergeability test is **cheap** using DBMs
 - Partially based on a purely **syntactic criterion**
 - However different ways of merging sets of mergeable DBMs



- But **efficient heuristics** proposed [David, 2005]

Extending Merging to Parametric State Automata

- Problems when extending the principle of merging to parametric timed automata
 - 1 Constraints are now constraints over the clocks and the parameters
 - 2 What properties are preserved when merging states?
 - 3 Implementation issues
 - No structure equivalent to DBMs exists for parametric timed automata (parametric DBMs [Hune et al., 2002] are not as efficient as DBMs)
 - ↪ No syntactic criterion

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Principle

We use the same definition as for timed automata

Definition

Two states $(\bullet, C_1)$ and $(\bullet, C_2)$ are **mergeable** if

- 1 Their discrete part is equal ($\bullet = \bullet$), and
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Their **merging** is $(\bullet, C_1 \cup C_2)$.

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- Breadth-first analysis
 - At each level n , compute the successor states of level $n + 1$

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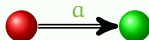
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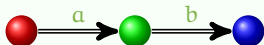
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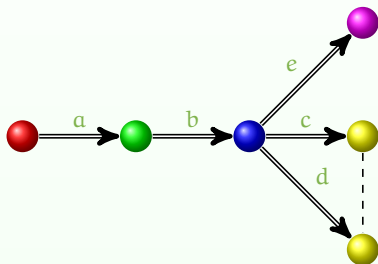
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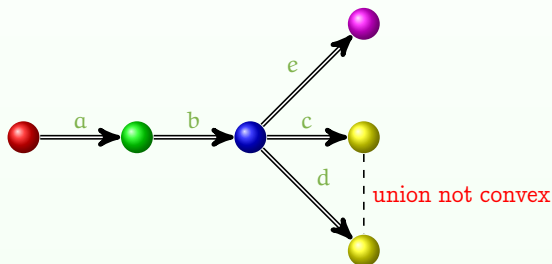
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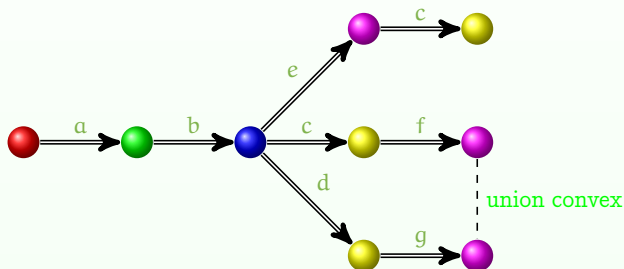
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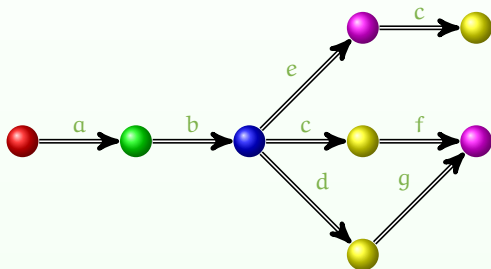
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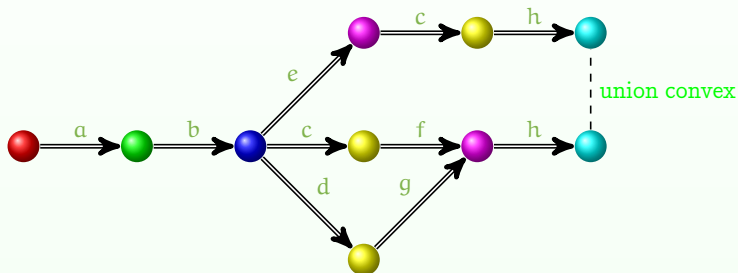


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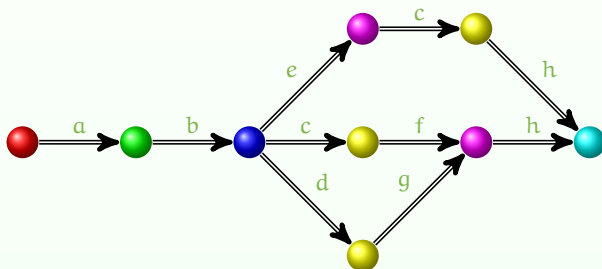
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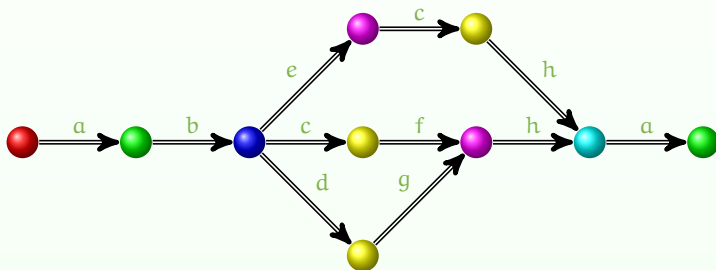
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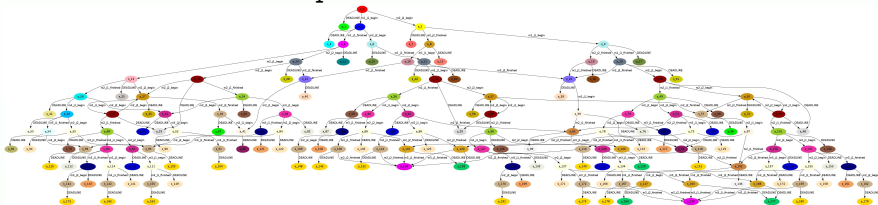
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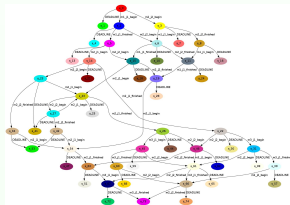


A Graphical Example

- The trace set of an example PTA



- The trace set obtained from the same PTA when merging states at each level



Case study "LA02" (trace sets generated by IMITATOR)

Properties

Notations:

- $Paths(\mathcal{A})$: paths of $\mathcal{A}$
- $Paths_{Mrg}(\mathcal{A})$: paths of $\mathcal{A}$ obtained by merging states at each level

Theorem (Characterization of merging)

- For all $(\bullet_0, C_0) \xRightarrow{a_0} \dots \xRightarrow{a_{n-1}} (\bullet_n, C_n) \in Paths(\mathcal{A})$, there exist $C'_1, \dots, C'_n$ such that:
 - 1 $(\bullet_0, C'_0) \xRightarrow{a_0}_{Mrg} \dots \xRightarrow{a_{n-1}}_{Mrg} (\bullet_n, C'_n) \in Paths_{Mrg}(\mathcal{A})$, and
 - 2 $C_i \subseteq C'_i$, for all $0 \leq i \leq n$.
- For all $(\bullet, C) \in Reach^*_{Mrg}(\mathcal{A})$ there exist $m \in \mathbb{N}$ and $(\bullet, C_1), \dots, (\bullet, C_m) \in Reach^*(\mathcal{A})$ such that

$$C = \bigcup_{1 \leq i \leq m} C_i.$$

Properties: Interpretation

- 1 The **traces** of $\mathcal{A}$ are **included** in the traces of $\mathcal{A}$ with merging
 - Hence the set of traces is over-approximated when merging states
 - The converse is not true in general
- 2 Each **time-abstract transition** $\bullet \xrightarrow{a} \bullet$ in the traces of $\mathcal{A}$ exists in the traces of $\mathcal{A}$ with merging, and conversely
 - Cannot be generalized to full traces!
- 3 The **set of parameter valuations allowing to reach a location** $\bullet$ in $Paths(\mathcal{A})$ is the same as the set of parameter valuations allowing to reach $\bullet$ in $Paths_{Mrg}(\mathcal{A})$

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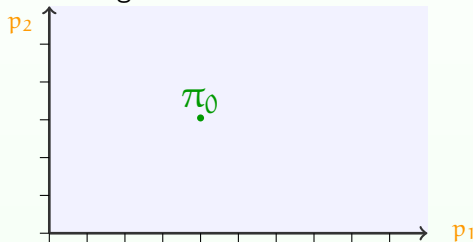
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Consequence

Merging states is safe for safety but not for linear-time properties.

The Inverse Method *IM*

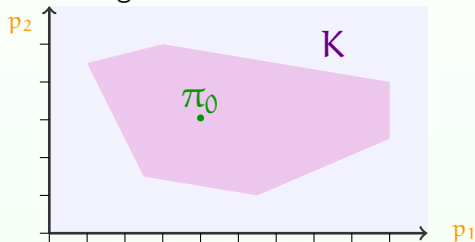
- Algorithm for the synthesis of parameters in PTA
[A., Chatain, Encrenaz, Fribourg, 2009]
- Takes as input a **reference parameter valuation** π_0 , and outputs a **constraint** K generalizing this reference valuation



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The Inverse Method: Example

 $\pi_0 :$

$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

 $K = \text{true}$

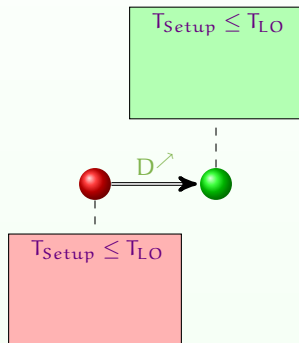

$$T_{Setup} \leq T_{LO}$$

Example of a flip-flop circuit [Clarísó and Cortadella, 2007]

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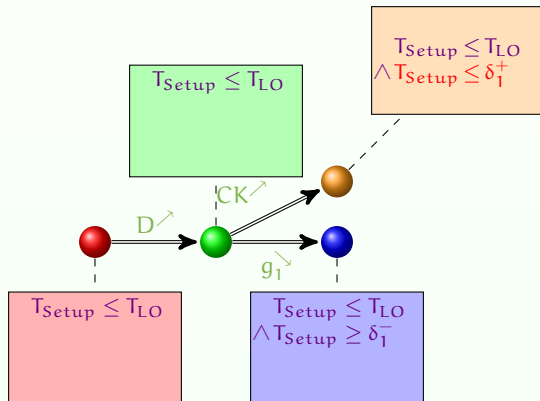
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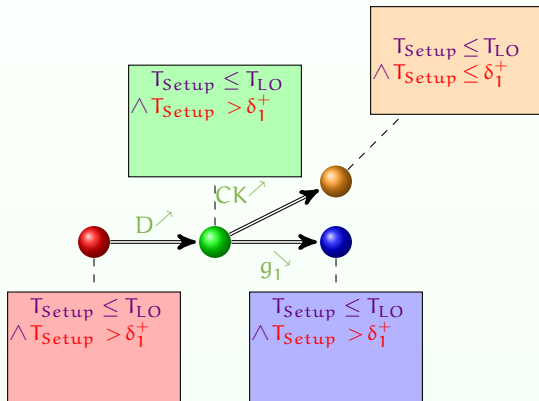
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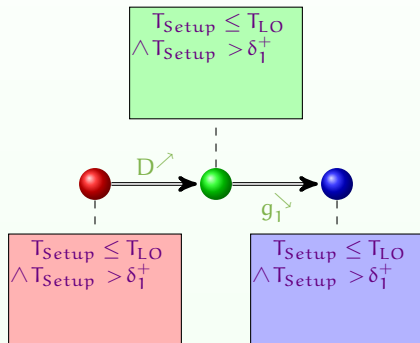
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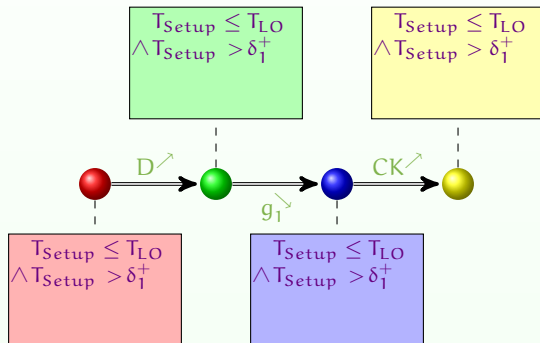
The Inverse Method: Example

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$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

 $K =$

$$T_{Setup} > \delta_1^+$$



Example of a flip-flop circuit [Clarísó and Cortadella, 2007]

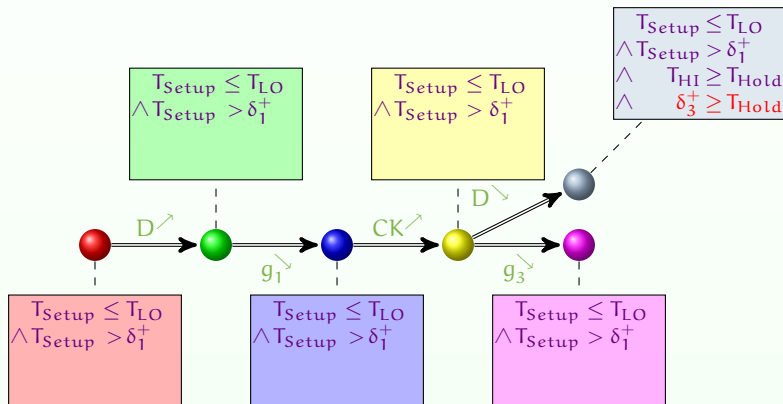
The Inverse Method: Example

 $\pi_0 :$

$\delta_1^- = 7$	$\delta_1^+ = 7$	$T_{HI} = 24$
$\delta_2^- = 5$	$\delta_2^+ = 6$	$T_{LO} = 15$
$\delta_3^- = 8$	$\delta_3^+ = 10$	$T_{Setup} = 10$
$\delta_4^- = 3$	$\delta_4^+ = 7$	$T_{Hold} = 17$

 $K =$

$$T_{Setup} > \delta_1^+$$



Example of a flip-flop circuit [Clarisó and Cortadella, 2007]

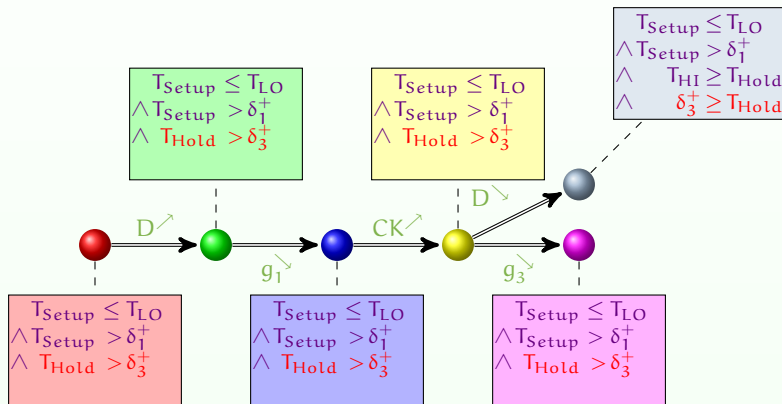
The Inverse Method: Example

 $\pi_0 :$

$$\begin{array}{lll} \delta_1^- = 7 & \delta_1^+ = 7 & T_{HI} = 24 \\ \delta_2^- = 5 & \delta_2^+ = 6 & T_{LO} = 15 \\ \delta_3^- = 8 & \delta_3^+ = 10 & T_{Setup} = 10 \\ \delta_4^- = 3 & \delta_4^+ = 7 & T_{Hold} = 17 \end{array}$$

 $K =$

$$\begin{array}{l} T_{Setup} > \delta_1^+ \\ \wedge T_{Hold} > \delta_3^+ \end{array}$$



Example of a flip-flop circuit [Clarisó and Cortadella, 2007]

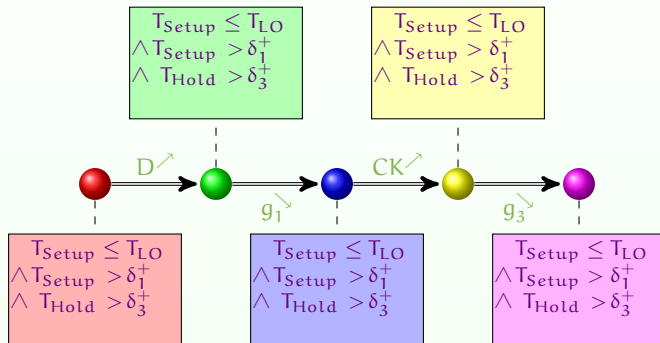
The Inverse Method: Example

 $\pi_0 :$

$$\begin{array}{lll} \delta_1^- = 7 & \delta_1^+ = 7 & T_{HI} = 24 \\ \delta_2^- = 5 & \delta_2^+ = 6 & T_{LO} = 15 \\ \delta_3^- = 8 & \delta_3^+ = 10 & T_{Setup} = 10 \\ \delta_4^- = 3 & \delta_4^+ = 7 & T_{Hold} = 17 \end{array}$$

 $K =$

$$\begin{array}{l} T_{Setup} > \delta_1^+ \\ \wedge T_{Hold} > \delta_3^+ \end{array}$$



Example of a flip-flop circuit [Clarísó and Cortadella, 2007]

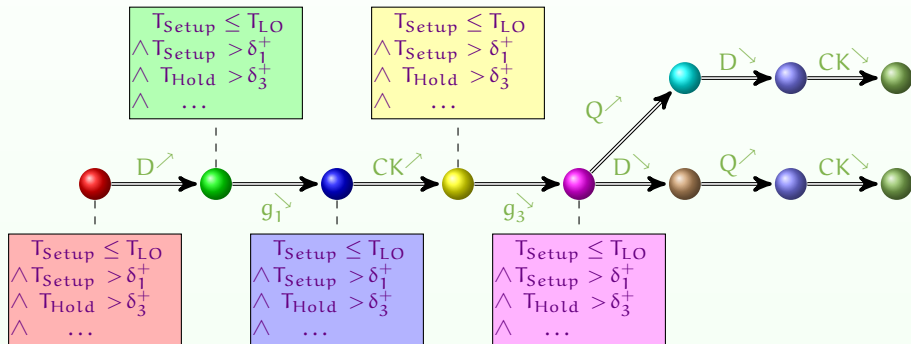
The Inverse Method: Example

 $\pi_0 :$

$$\begin{array}{lll} \delta_1^- = 7 & \delta_1^+ = 7 & T_{HI} = 24 \\ \delta_2^- = 5 & \delta_2^+ = 6 & T_{LO} = 15 \\ \delta_3^- = 8 & \delta_3^+ = 10 & T_{Setup} = 10 \\ \delta_4^- = 3 & \delta_4^+ = 7 & T_{Hold} = 17 \end{array}$$

 $K =$

$$\begin{array}{l} T_{Setup} > \delta_1^+ \quad \wedge \quad \delta_3^+ + \delta_4^+ \geq T_{Hold} \\ \wedge \quad T_{Hold} > \delta_3^+ \quad \wedge \quad \delta_3^+ + \delta_4^+ < T_{HI} \\ \wedge \quad T_{Setup} \leq T_{LO} \quad \wedge \quad \delta_3^- + \delta_4^- \leq T_{Hold} \\ \wedge \quad \delta_1^- > 0 \end{array}$$



Example of a flip-flop circuit [Clarísó and Cortadella, 2007]

The Inverse Method

Starting from the initial state with $K = \text{true}$

While there are successor states **do**

- 1 Compute successor states (breadth-first)
- 2 **if** K is false **then** **return** false **endif**
- 3 Remove π_0 -incompatible states ($\bullet, C$) (i.e., such that $\pi_0 \not\models C$) by refining K
- 4 Go to 1

Return the intersection of all constraints

The Inverse Method With Merging IM_{Mrg}

Starting from the initial state with $K = \text{true}$

While there are successor states **do**

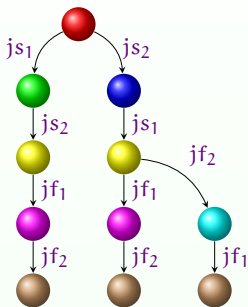
- 1 Compute successor states (breadth-first)
- 2 Merge these successor states
- 3 Remove π_0 -incompatible states ($\bullet, C$) (i.e., such that $\pi_0 \not\models C$) by refining K
- 4 Go to 1

Return the intersection of all constraints

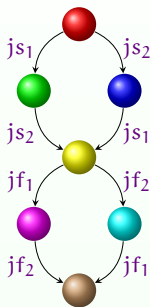
IM_{Mrg} : The Problem

■ Example of a jobshop case study

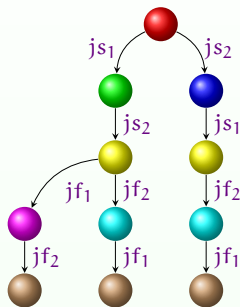
For a given π_0



For $IM_{Mrg}(\mathcal{A}, \pi_0)$



For some $\pi \models IM_{Mrg}(\mathcal{A}, \pi_0)$



■ Problems

- Trace sets are not equal
- Equality of locations but no equality of actions in general

IM_{Mrg} : Properties

Theorem

Suppose $IM_{Mrg}(\mathcal{A}, \pi_0)$ terminates with output K_{Mrg} . Then

- 1 $\pi_0 \models K_{Mrg}$, and
- 2 For all $\pi \models K_{Mrg}$, the reachable *locations* are the same for π_0 and π .

IM_{Mrg} : Properties

Theorem

Suppose $IM_{Mrg}(\mathcal{A}, \pi_0)$ terminates with output K_{Mrg} . Then

- 1 $\pi_0 \models K_{Mrg}$, and
- 2 For all $\pi \models K_{Mrg}$, the reachable *locations* are the same for π_0 and π .

For *backward-deterministic* PTA, equality of the actions set is guaranteed; but not for general PTA.

Inverse Method With Merging

Starting from the initial state with $K = \text{true}$

While there are successor states **do**

- 1 Compute successor states (breadth-first)
- 2 Merge these successor states
- 3 Remove π_0 -incompatible states ($\bullet, C$) (i.e., such that $\pi_0 \not\models C$) by refining K
- 4 Go to 1

Return the intersection of all constraints

An Improved Inverse Method With Merging IM'_{Mrg}

Starting from the initial state with $K = \text{true}$

While there are successor states **do**

- 1 Compute successor states (breadth-first)
- 2 Remove π_0 -incompatible states ($\bullet, C$) (i.e., such that $\pi_0 \not\models C$) by refining K
- 3 Merge these successor states
- 4 Go to 1

Return the intersection of all constraints

IM'_{Mrg} : Properties

Theorem

Suppose $IM'_{Mrg}(\mathcal{A}, \pi_0)$ terminates with output K'_{Mrg} . Then

- 1 $\pi_0 \models K'_{Mrg}$, and
- 2 For all $\pi \models K'_{Mrg}$, the reachable *locations* and executable *actions* are the same for π_0 and π .

Remarks

- Equality of trace sets is still not guaranteed
- We have $K \subseteq K'_{Mrg} \subseteq K_{Mrg}$
 - IM'_{Mrg} can be seen as a *tradeoff* between IM without merging (the state space of which may blow up), and IM_{Mrg} (that does not preserve actions)

Outline

- 1 Parametric Timed Automata
- 2 Merging States in Timed Automata
- 3 Merging States in Parametric Timed Automata
- 4 Experiments**
- 5 Perspectives

Implementation in IMITATOR

- IMITATOR 2.6 [A., Fribourg, Kühne, Soulat, 2012]
 - “Inverse Method for Inferring Time Abstract Behavior”
 - 10,000 lines of OCaml code
 - Relies on the PPL library for operations on polyhedra [Bagnara et al., 2008]
 - Available under the GNU-GPL license
 - Now integrated in the *CosyVerif* platform [AHHKLLP13]
- Experimental validation by comparing performances when executing IM and IM'_{Mrg}

Experiments

Example	X	P	<i>IM</i>				<i>IM'</i> _{Mrg}				Comparison		
			States	Trans.	t	Cpl	States	Trans.	t	Cpl	States	t	K
AndOr	4	12	11	11	0.052	✓	9	9	0.056	✓	82	108	=
Flip-Flop	5	12	11	10	0.060	✓	9	9	0.057	✓	82	108	=
Latch	8	13	18	17	0.083	?	12	12	0.069	?	67	83	=
SPSMALL	10	26	31	30	0.618	?	31	30	0.613	?	100	99	=
SIMOP	8	7	-	-	loop	-	172	262	2.52	?	0	0	-
BRP	7	6	429	474	3.50	✓	426	473	4.30	✓	99	123	=
CSMA/CD	3	3	301	462	0.514	✓	300	461	0.574	✓	100	112	=
CSMA/CD'	3	3	13,365	14,271	18.3	✓	13,365	14,271	25.4	✓	100	139	=
RCP	5	6	327	518	0.748	✓	115	186	0.684	✓	35	91	=
WLAN	2	8	-	-	loop	-	8,430	15,152	2,137	✓	0	0	-
ABT	7	7	63	62	0.344	?	63	62	0.335	?	100	97	=
AM02	3	4	182	215	0.369	✓	53	70	0.112	✓	29	30	⊂
BB04	6	7	806	827	28.0	?	141	145	3.15	?	17	11	=
CTC	15	21	1,364	1,363	88.9	✓	215	264	17.6	✓	16	20	=
LA02	3	5	6,290	8,023	751	?	383	533	17.7	✓	6.0	2.4	⊂
LPPRC10	4	7	78	102	0.39	?	31	40	0.251	?	40	64	=
M2.4	3	8	1,497	1,844	8.89	✓	119	181	0.374	✓	7.9	4.2	⊂

Sources: <http://www.lsv.ens-cachan.fr/Software/imitator/merging/>

Experiments: Remarks

- From our results, IM'_{Mrg} always has a number of states equal to or less than IM
 - Strictly less states in all but 4 experiments
 - Very efficient for scheduling problems: up to a division by 16
 - Some case studies can only be verified using IM'_{Mrg}
- Time reduction
 - When no states are merged, up to 39 % extra time
 - Mergeability test expensive using polyhedra
 - Reasonable?
 - More surprisingly, despite expensive overhead, IM'_{Mrg} is often faster than IM
 - Up to a division by 42
- Set of parameter valuations output by IM'_{Mrg} often larger than IM

Outline

- 1 Parametric Timed Automata
- 2 Merging States in Timed Automata
- 3 Merging States in Parametric Timed Automata
- 4 Experiments
- 5 Perspectives**

Conclusion

- Characterization of **states merging** for parametric timed automata
 - Safety properties preserved, but not linear-time properties
- Characterization of **states merging** in the **inverse method**
 - No preservation of the trace set
 - Locations preserved, but not actions in general
 - Improvement to preserve actions too
- Experimental validation
 - Improvement of both **memory and time** for many case studies
 - Reasonable time overhead in worst case studies
 - **Larger set of parameter valuations** synthesized

Perspectives

- Improve the heuristics
 - When to merge?
 - May change the properties of merging
 - On which states should the mergeability test be applied?
 - Expensive test to be performed only when its has good chances to be positive
- Combine with other state space reduction techniques
 - In particular (quasi-)equal clock elimination
- Extend to larger classes of models such as hybrid systems
[Alur et al., 1993a, Fribourg and Kühne, 2013]

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Additional explanation

The Inverse Method: Algorithm

Algorithm 1: $IM(\mathcal{A}, \pi)$

input : PTA $\mathcal{A}$ of initial state s_0 , parameter valuation π

output: Constraint K over the parameters

```

1  $i \leftarrow 0$ ;  $K_c \leftarrow \text{true}$ ;  $S_{new} \leftarrow \{s_0\}$ ;  $S \leftarrow \{\}$ 
2 while  $\text{true}$  do
3   while there are  $\pi$ -incompatible states in  $S_{new}$  do
4     Select a  $\pi$ -incompatible state  $(l, C)$  of  $S_{new}$  (i.e., s.t.  $\pi \not\models C$ ) ;
5     Select a  $\pi$ -incompatible  $J$  in  $C \downarrow_P$  (i.e., s.t.  $\pi \not\models J$ ) ;
6      $K_c \leftarrow K_c \wedge \neg J$ ;  $S \leftarrow \bigcup_{j=0}^{i-1} Post_{\mathcal{A}(K_c)}^j(\{s_0\})$ ;  $S_{new} \leftarrow Post_{\mathcal{A}(K_c)}(S)$  ;
7   if  $S_{new} \sqsubseteq S$  then return  $K \leftarrow \bigcap_{(l, C) \in S} C \downarrow_P$ 
8    $i \leftarrow i + 1$ ;  $S \leftarrow S \cup S_{new}$ ;  $S_{new} \leftarrow Post_{\mathcal{A}(K_c)}(S)$ 

```

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